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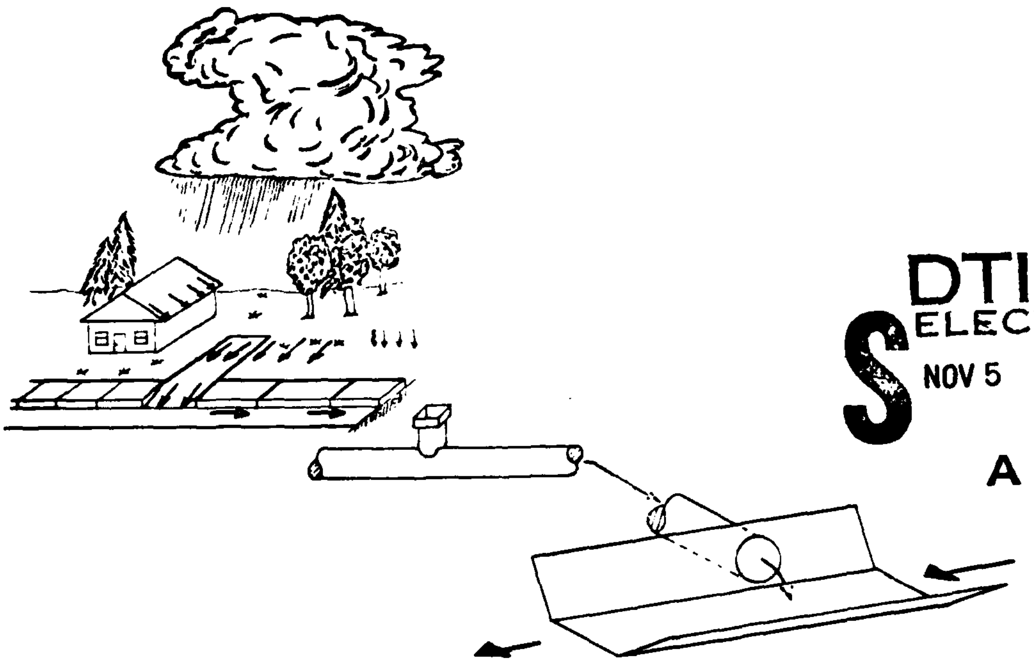
INTRODUCTION AND APPLICATION OF KINEMATIC WAVE ROUTING TECHNIQUES USING HEC-1

Training Document No. 10



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This document discusses the application of the kinematic wave routing method in HEC-1 for analyzing urban runoff processes. The physical processes of the urban runoff and streamflow routing are discussed briefly and related to the kinematic wave capabilities in the HEC-1 Flood Hydrograph Package. Date requirements along with specific methods of applying kinematic wave routing techniques to runoff problems in urban hydrology and example applications of the method to analyze typical problems are discussed.
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Chapter 1 presents introductory material necessary for an understanding of the theory, assumptions, equations and numerical methods incorporated into HEC-1 for kinematic wave flood routing. Chapter 2 explains methods of applying kinematic wave routing techniques using HEC-1. An example problem is presented to illustrate HEC-1 input and output data, and effects of changes to numerical values of the parameters are discussed. Results of a "hand" calculation are given in Appendix A to illustrate the basic solution procedure.

B

INTRODUCTION AND APPLICATION OF KINEMATIC WAVE ROUTING TECHNIQUES USING HEC-1

Training Document No. 10

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MAY 1979

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INTRODUCTION AND APPLICATION OF KINEMATIC WAVE
ROUTING TECHNIQUES USING HEC-1

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FOREWORD

This document discusses the application of the kinematic wave routing method in HEC-1 for analyzing urban runoff processes. The physical processes of the urban runoff and streamflow routing are discussed briefly and related to the kinematic wave capabilities in the HEC-1 Flood Hydrograph Package. Data requirements along with specific methods of applying kinematic wave routing techniques to runoff problems in urban hydrology and example applications of the method to analyze typical problems are discussed.

Chapter 1 of this document presents introductory material necessary for an understanding of the theory, assumptions, equations and numerical methods incorporated into HEC-1 for kinematic wave flood routing. Chapter 2 explains methods of applying kinematic wave routing techniques using HEC-1. The chapters have been prepared so they stand separately and either one or both may be read. A reader interested only in the theory, or only in application procedures, can read the chapter desired.

This newly added procedure has had only limited use to date at the Hydrologic Engineering Center (HEC), although the general method has been used with success in a number of other hydrologic models.

Readers interested in explanations with greater detail should refer to the specific references listed at the end of each chapter.

DEFINITION OF TERMS

TERM	DEFINITION	DIMENSIONS
A	cross sectional area of flow, Equation (1.7)	ft^2
A_c	cross sectional area of flow for collector channel elements, Equations (1.13), (1.14), (1.24)	ft^2
A_m	cross sectional area of flow for main channel and stream elements	ft^2
A_{BASIN}	total surface area of the basin; A $A_{BASIN} = \Sigma A_{subbasin}$	mi^2
$A_{subbasin}$	surface area of a subbasin	mi^2
A_p	percentage of the subbasin area which would be pervious	%
A_i	percentage of the subbasin area which would be impervious	%
c	shallow wave celerity, $c = \sqrt{gD}$	ft/sec
	(subscript): indicates properties pertaining to the collector channel elements	N-D*
D	hydraulic depth of channel	ft
D_c	diameter of circular collector channel Figure 1.7	ft
f	infiltration rate, Equations (1.1), (1.11), (1.12)	cfs/ft^2
$\mathbf{F}$	Froude number $= V/\sqrt{gD}$, Equation (1.6)	N-D
g	acceleration of gravity, Equations (1.2), (1.4)	ft/sec^2
i	rainfall intensity, Equations (1.1), (1.11), (1.12)	
	(subscript): indicates spacial location on a finite difference grid, Figure 1.9	N-D

*N-D - non-dimensional

TERM	DEFINITION	DIMENSIONS
j	(subscript): indicates temporal location on a finite difference grid, Figure 1.9	N-D
k	dimensionless kinematic flow number, Equation (1.6).	N-D
L	channel length, length of overland flow plane, Equation (1.6)	ft
	(subscript): to indicate lateral inflow, Equations (1.1), (1.2), (1.4)	N-D
L_o	length of overland flow element, Figure 1.3	ft
L_c	length of collector element, Figures 1.3, 1.4	ft
L_m	length of main channel element, Figures 1.3, 1.4	ft
L_{o_1}	length of type 1 overland flow element, Figure 1.4	ft
L_{o_2}	length of type 2 overland flow element, Figure 1.4	ft
m	(subscript): indicates properties pertaining to main channel elements, Figures 1.3, 1.4	N-D
m	kinematic routing coefficient, Equation (1.3)	N-D
m_o	kinematic routing coefficient for overland flow elements, Equations (1.10), (1.12)	N-D
m_c	kinematic routing coefficient for collector channel elements, Equations (1.14), (1.19), (1.20), (1.21), (1.22), (1.23), (1.24)	N-D
n	Manning's resistance coefficient, Equation (1.7)	$sec/ft^{1/3}$
N	effective roughness parameter for overland flow, Equation (1.8), Table 2.1	$sec/ft^{1/3}$
o	(subscript): indicates properties pertaining to overland flow elements	N-D

TERM	DEFINITION	DIMENSIONS
P	wetted perimeter	ft
q	discharge per unit width of channel, Equations (1.1), (1.9)	cfs/ft
q_c	discharge per unit width of collector channel, Figures 1.3, 1.4	cfs/ft
q_o	discharge per unit width of overland flow element, Figures 1.3, 1.4	cfs/ft
q_L	total lateral inflow per unit length of channel	cfs/ft^2
q_{o_1}	discharge per unit width of overland flow strip from type 1 flow elements, Figure 1.4	cfs/ft
q_{o_2}	discharge per unit width of overland flow strip from type 2 flow elements, Figure 1.4	cfs/ft
Q	discharge, Equations (1.3), (1.4), (1.7), (1.8)	cfs
Q_{OUT}	total discharge from a subbasin, Figures 1.3, 1.4	cfs
Q_*	dimensionless discharge, Figure 1.6	N-D
R	hydraulic radius - A/P, Equation (1.7)	ft
S_f	friction slope defined by Manning's equation, Equation (1.1)	N-D
S_o	average bottom slope, Equations (1.2), (1.4), (1.6), (1.7), Figure 1.3	N-D
S_c	average bottom slope for main collector channel elements, Equations (1.15), (1.19), Figure 1.3	N-D
S_m	average bottom slope for main channel elements, Figure 1.3	N-D
t	time, Figures 1.8, 1.9, Equations (1.1), (1.2), (1.11), (1.13), (1.29)	sec
t_*	dimensionless time, Figure 1.6	N-D
Δt	time step used in finite difference equations, Equations (1.29), (1.31), (1.32), Figures 1.8, 1.9	sec

TERM	DEFINITION	DIMENSIONS
u	x-component of mean velocity, Equations (1.2), (1.4)	ft/sec
v	x-component of velocity from lateral inflow into a channel (assumed negligible), Equation (1.2)	ft/sec
V	mean cross section velocity = Q/A	ft/sec
w	bottom width of typical collector or main channel, Figure 1.7	ft
x	longitudinal distance, Equations (1.1), (1.2) spacial direction, Figures 1.8, 1.9	ft
Δx	spacial stepping distance used in finite difference equations, Equations (1.25), (1.28), Figures 1.8, 1.9	ft
y	mean depth in St. Venant equation, Equations (1.1), (1.2), (1.4)	ft
y_c	mean depth of flow in collector channel elements, Equations (1.16), (1.17), Figure 1.7	ft
z	side slope for generalized trapezoidal cross section used for collector or main channel routing, Equations (1.16), (1.17), (1.18), (1.19), (1.22), Figure 1.7	N-D
α	kinematic wave routing parameter for a particular cross sectional shape, slope and roughness, Equations (1.3), (1.10), (1.14), (1.19), (1.20), (1.21), (1.22), (1.23)	

CHAPTER 1

AN INTRODUCTION TO KINEMATIC FLOW APPROXIMATIONS

by Robert C. MacArthur[1]

INTRODUCTION

Rapid urbanization of river basins in and around metropolitan areas has forced land and water resources planners and hydrologists to develop a variety of methods for analyzing problems in urban hydrology. Problems involving both design and management decisions are often so complex as to require application of mathematical models. The mathematical models which have been most commonly used rely on basic unit-graph techniques to model the application and distribution of precipitation as rainfall or snowmelt, compute rainfall and snowmelt losses and excesses, and determine subbasin outflow hydrographs. Although these models which are based on the development of a representative unit hydrograph are frequently applied and have been used successfully, it is difficult to associate physical properties of the basin to be modeled to the parameters necessary to develop a unit hydrograph. It becomes even more difficult to define some of the parameters such as the Clark T_c and R or Snyder C_t and C_p for basins which have no recorded data. Because it is most important to develop the best representation of the actual urban runoff situation when

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analyzing urban storm water runoff problems, it would be desirable to relate runoff processes directly to measurable geographic features of the basin. It would also be desirable for the modeling technique to be able to reproduce nonlinear runoff characteristics rather than being limited to linear responses such as those developed by unit graph techniques.

The kinematic wave method of routing overland and river flows has been chosen as an additional routing option for use in the HEC-1 Flood Hydrograph Package for several reasons. Although simple in form, kinematic wave theory offers the benefits of nonlinear response without needing an unduly complicated or costly solution procedure. In addition, for the purposes of modeling unsteady overland flow, any model will require considerable parameter adjustment to account for the complexities of the basin and the specific flows which occur within the basin. The kinematic wave method relates basin and flow characteristics directly to the two routing parameters, α and m. The parameters α and m are directly related to the shape of the channel, the boundary roughness and the slope of the channel or overland flow surface. There also have been several previous studies which have developed sets of appropriate values for these parameters for a large range of flow and boundary conditions. Numerical techniques used to simulate overland and river flows can only approximate the actual response of real systems because of the complex nature of natural drainage basins and because simplifications must be made to the mathematics to make the model efficient and economical to execute. The kinematic wave approximation has been proven to be an accurate

and efficient method of simulating stormwater runoff from small basins for both overland flow and stream channel routing (Overton and Meadows, 1976).

MODELING UNSTEADY FLOW USING THE KINEMATIC WAVE APPROACH

Kinematics is defined as the study of motion exclusive of the influences of mass and force, in contrast with dynamics, in which these influences are included. Flood waves can be identified as either of two separate kinds of wave phenomena: the dynamic wave and the kinematic wave. Although both of these kinds of waves are initially present, certain characteristics of a watershed can make kinematic waves the dominant characteristic of a flood event.

When inertial and pressure forces are important, "dynamic waves" govern the movement of long waves in shallow water, like a large flood wave in a wide river (Stoker, 1957). When the inertial and pressure forces are not important to the movement of the wave, "kinematic waves" govern the flow. For this latter flow condition, the weight component (the force in the direction of the channel axis due to the weight of the fluid flowing downhill in response to the action of gravity) is approximately balanced by the resistive forces due to channel bed friction (in most cases this is represented by Manning's equation). Flows of this nature (kinematic waves) will not be accelerating appreciably and the flow will remain approximately uniform along the channel. No visible surface wave will be noticeable and the passage of the flood wave, as depicted in Figure 1.1, will be

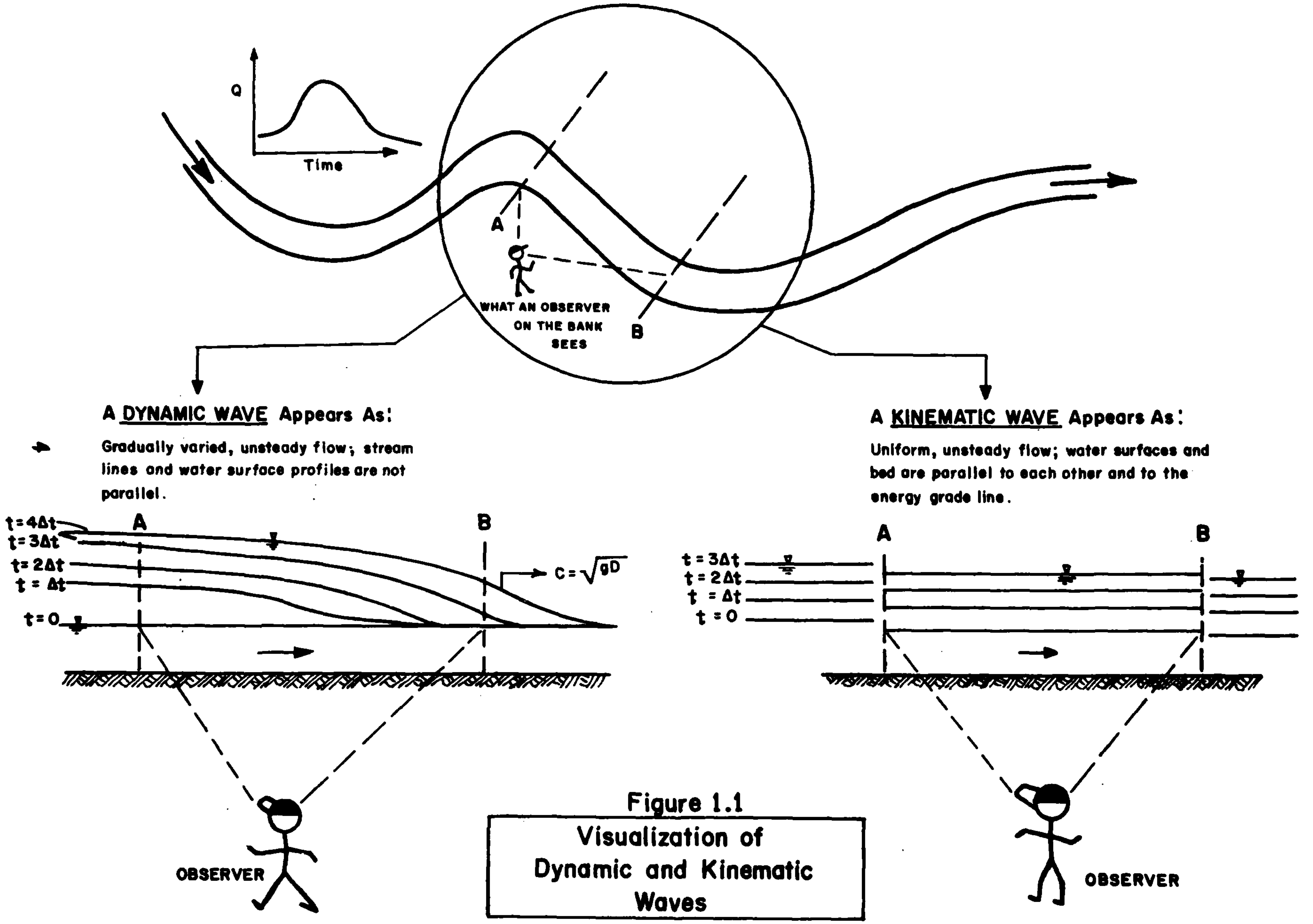

Figure 1.1
Visualization of Dynamic and Kinematic Waves

seen by an observer on the bank as an apparently unifrom rise and fall in the water surface elevation over a relatively long period of time with respect to the size of the specific subbasin being analyzed. Therefore, kinematic flows are often classified as uniform unsteady flows.

Lighthill and Whitham (1955) found that because dynamic waves normally have much higher velocities and attenuate more quickly than kinematic waves, flood flows are generally dominated by kinematic waves. Even though any surface disturbance will send a "signal" downstream at the speed of small gravity waves, this "signal" will be too weak to be deteced at any considerable distance downstream. Therefore, a main flow change or "signal" is carried as a kinematic wave at much slower velocities and the speed of flood waves may be approximated by the speed of kinematic waves. In this context, a kinematic wave represents the characteristic changes in discharge, velocity and water surface elevation with time as observed at any one location on an overland flow plane or along a stream channel.

The speed of small gravity waves occurring in shallow open channels is often referred to as wave celerity, c, and is equal to $\sqrt{gD}$, where D is the channel hydraulic depth (Chow, 1959). The ratio of the fluid speed (the mean cross-sectional velocity) to the celerity is called the Froude number, $\mathbf{F}$. Therefore, the Froude number also represents the ratio of inertial forces to gravity forces.

Flows with Froude numbers greater than one are classified as supercritical flows (see Chow, 1959), and surface waves are unable to move in the upstream direction because the flow velocity $V = Q/A$

is greater than the wave celerity, $c = \sqrt{gD}$. Flows with Froude numbers greater than two tend to be unstable, which may affect the accuracy and applicability of steady flow assumptions for high Froude number flows. Lighthill and Whitham (1955), however, found that the characteristics of kinematic waves dominate over those of dynamic waves for flows with Froude numbers that are less than or equal to two (e.g., $\mathbf{F} \leq 2$). In fact, they found that for $\mathbf{F} < 1$, dynamic waves decay exponentially with respect to a time constant they chose to define as $V/[gS(1-\mathbf{F}/2)]$, where, S, is the channel or surface runoff slope. Therefore, one may conclude that kinematic waves will ultimately dominate the flow characteristics occurring for overland flows and small watershed channel flows when the flow Froude number is less than two.

This brief introduction to kinematic wave approximations leads to the following discussion and definition of uniform unsteady flows.

Review of the Basic Types of Flow

As a brief review, several basic types of open channel flow that are most commonly experienced will be discussed to remind the reader of some of the important differences and characteristics of flows found in practice (portions of this discussion can be found in Thomas, 1975). Figure 1.2, presents several possible types of open channel flow.

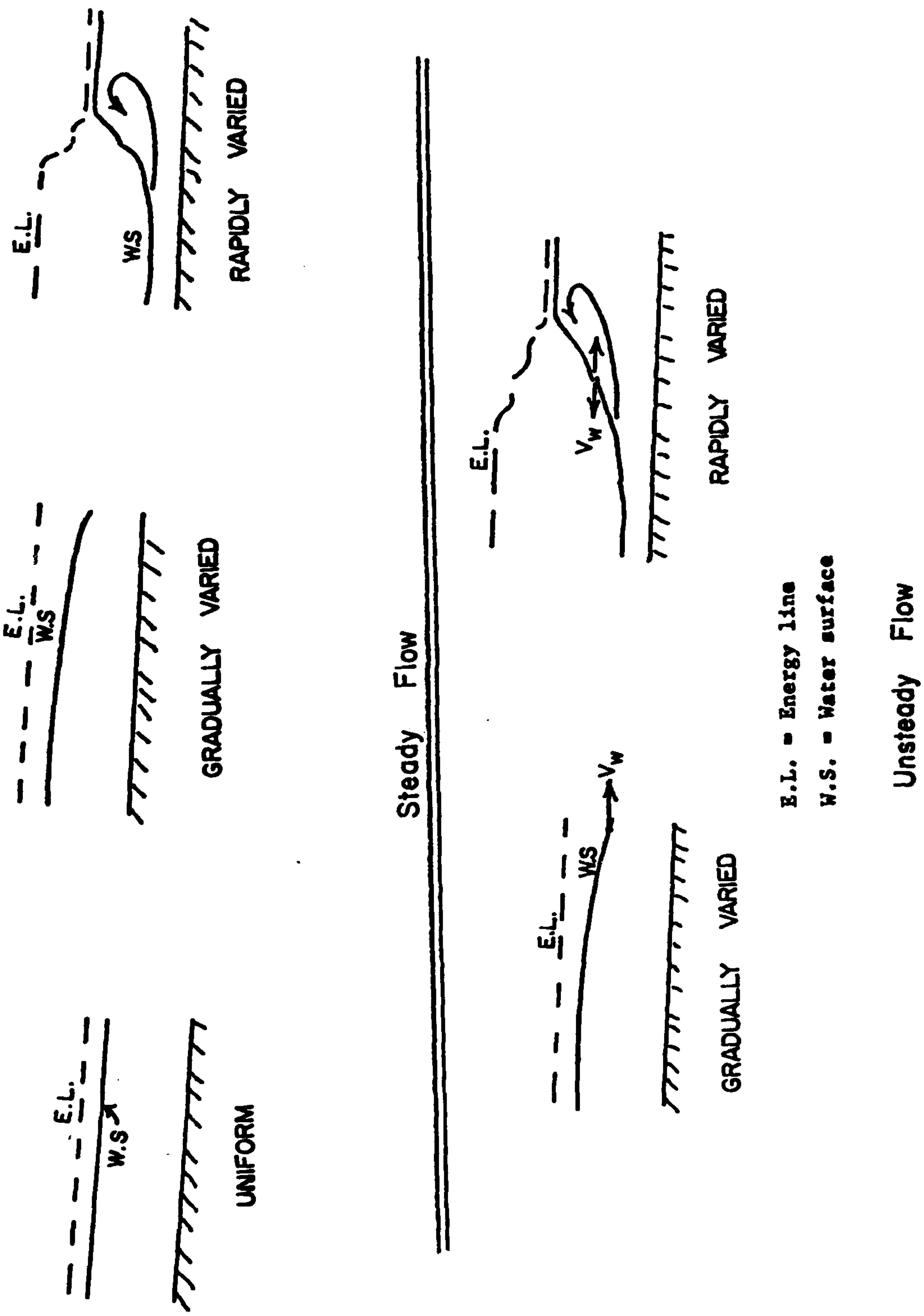

Figure 1.2 Possible Types of Open Channel Flow
(From Thomas, July 1975)

Steady versus Unsteady Flow. If the change in velocity with respect to time at a given location is zero, the flow is called steady flow. Otherwise, the flow is classified as unsteady flow. Therefore, unsteady flows require the consideration of time as an additional variable.

Uniform versus Varied Flow. If the change in channel flow velocity, V, with respect to distance along the channel, x, is zero (i.e., $dV/dx = 0$) for a given period of time, the flow is called uniform. Otherwise, the flow is nonuniform and the relationship between kinetic energy and potential energy will be changing along the channel. When the flow is uniform, the water surface will be parallel to the channel bottom. If the flow is not uniform, the channel slope will be slightly different from the slope of the water surface.

If the rate of change of the water surface slope is not visible to the eye, the flow may be considered as gradually varied. Rapidly varied flows demonstrate apparent and rather large water surface slope changes (such as at a hydraulic jump). Rapidly varied flow requires special treatment and will not be considered herein.

DEVELOPMENT OF GOVERNING EQUATIONS

Although the basic differential equations capable of describing one dimensional gradually varied unsteady flow were originally developed a century ago, they have only been recently applied (within the last thirty to forty years) to general hydrologic engineering problems because it was not possible to solve these equations efficiently

without a high speed computer. Keulegan (1944) was probably the first to apply these techniques to the general problem of overland flow.

The mechanics of unsteady open channel flow may be expressed mathematically in terms of the equations developed in 1870 by St. Venant. These equations [Equations (1.1) and (1.2)] are partial differential equations which may be derived from the basic principles of conservation of mass and momentum. Various derivations have been presented and the reader is referred to the following references for detailed descriptions: Chow (1959), Henderson (1966), Strelkoff (1969), or Fread (1976).

St. Venant Equations

Continuity $$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = q_L + (i - f) \quad \text{(channels of unit width)} \qquad (1.1)$$

Momentum $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial y}{\partial x} = g(S_o - S_f) - q_L\frac{(u-v)}{y} \qquad (1.2)$$

where:

g = acceleration of gravity (ft/sec^2)
y = mean depth (ft)
q = discharge per unit width of channel (cfs/ft)
x = distance measured in downstream flow direction (ft)
t = time (seconds)
u = x - component of mean velocity (ft/sec)
i = rainfall intensity (cfs/ft^2)
f = infiltration rate (cfs/ft^2)
S_o = average bottom slope (ft/ft)
S_f = friction slope defined by the Manning equation
q_L = total lateral inflow per unit length of channel (cfs/ft^2)
v = the x-component of velocity for lateral inflow. (This is assumed to be negligible to the total momentum balance for channel routing and is therefore zero).

The four terms in Equation (1.1) and the five terms in Equation (1.2), are known successively as:

- rate of rise term
- storage term
- lateral inflow per unit length
- intensity of excess rainfall

Terms in the Continuity Equation (1.1)

- acceleration term
- velocity head term
- depth taper term
- bed slope minus friction term
- lateral inflow term

Terms in the Momentum Equation (1.2)

Prior to presenting the details of the kinematic flow method that have been incorporated into the HEC-1 Flood Hydrograph Package, it is important to discuss the basic assumptions and requirements associated with the equations used for gradually varied unsteady flows. If these basic assumptions are <u>not</u> valid for the intended flow conditions, then alternate methods of simulating the flow should be sought. (For further definitions of terms, refer to the "Definition of Terms" on pages *vii* - *x*.)

General Assumptions

In the development of the general unsteady flow equations it is assumed that the flow is one dimensional in the sense that flow characteristics such as depth and velocity are considered to vary only in the longitudinal x-direction of the channel. Additional basic assumptions necessary for the validity of the equations include:

(1) the velocity is constant and the water surface is horizontal across any section perpendicular to the longitudinal flow axis;

(2) all flows are gradually varied with hydrostatic pressure prevailing at all points in the flow such that all vertical accelerations within the water column can be neglected;

(3) the longitudinal axis of the flow channel can be approximated by a straight line, therefore, no lateral secondary circulations occur;

(4) the slope of the channel bottom is small (less than 1:10);

(5) the channel boundaries may be treated as fixed noneroding and nonaggrading;

(6) resistance to flow may be described by empirical resistance equations such as the Manning or Chezy Equations;

(7) momentum carried to the fluid from lateral inflows is negligible; and,

(8) the flow is incompressible and homogeneous in density.

The assumptions found in items 1 through 6 and 8 have been shown (Strelkoff, 1969 and others) to be valid and applicable for most open channel flows occurring in natural rivers and streams. The validity of the restrictions presented in assumption 5 above are not easily evaluated. Research is currently under way to estimate the overall affects of this assumption. It has also been found that overland flows such as those associated with stormwater runoff can be described by the kinematic wave form of Equations (1.1) and (1.2) without violation of the eight previous assumptions (Lighthill and Whitham, 1955; Liggett and Woolhiser, 1967). In most cases assumption number 7 will be valid. If, however, lateral flows are much more significant than the main channel flow then these local effects may require special attention and changes to methods currently used in the HEC-1 computer program.

DISCUSSION OF THE GENERAL CONCEPTS AND STRUCTURE OF THE KINEMATIC WAVE FLOOD ROUTING TECHNIQUE

A general conceptual description of the methods of modeling stormwater runoff using kinematic wave simplifications will be presented here prior to discussing the specific equations and algorithms that are used. This will help the reader understand how the model "visualizes" the actual physical characteristics of the basin and responds to rainfall and stormwater runoff. Specific mathematical relationships and numerical solution techniques will be presented in a following section.

Analyses of surface runoff problems are complicated not only by the nature of specific storm events which occur, but perhaps to a greater extent, by the nature and complexity of the watershed or urban area being analyzed. Description of the local physical characteristics, geometry, and response of the system could become a monumental task if one were to include every minute detail. The concept incorporated into the HEC-1 Flood Hydrograph Package involves simulating the natural complexities of a basin with a number of simple elements, such as overland flow planes, stream segments, and lengths of representative storm drain or sewer pipe as shown in Figures 1.3 and 1.4. If the previously listed gradually varied unsteady flow assumptions are not violated, then combinations of these basic elements have proven to be quite representative of the actual behavior of most systems. Wooding (1966) compared measured responses from three natural drainage areas with calculated results obtained from a model which used a simple kinematic flow routing procedure similar to that found in HEC-1. Wooding (1966) concluded that although the geometry of natural catchments

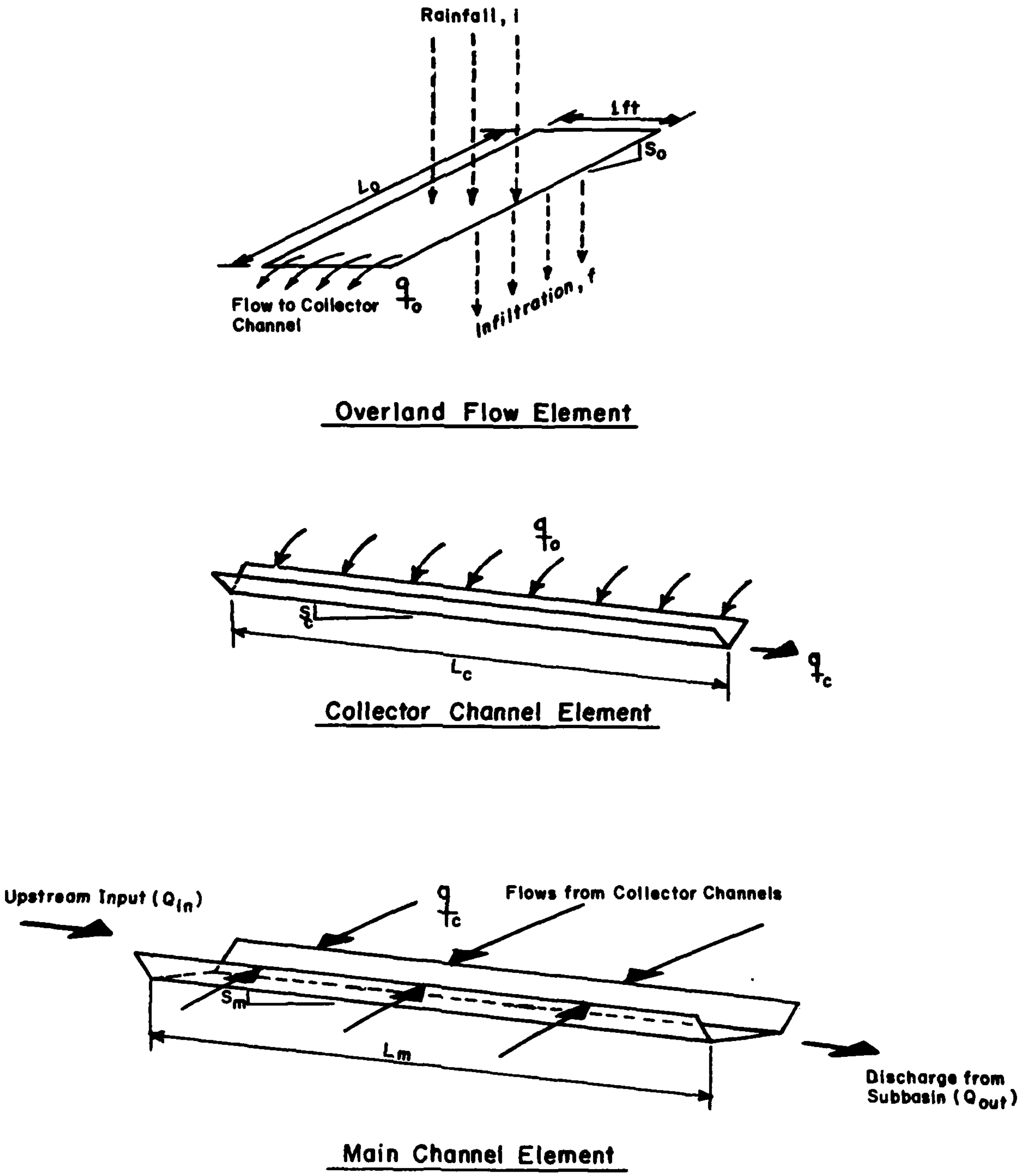

Figure 1.3 Elements Used in Kinematic Wave Calculations

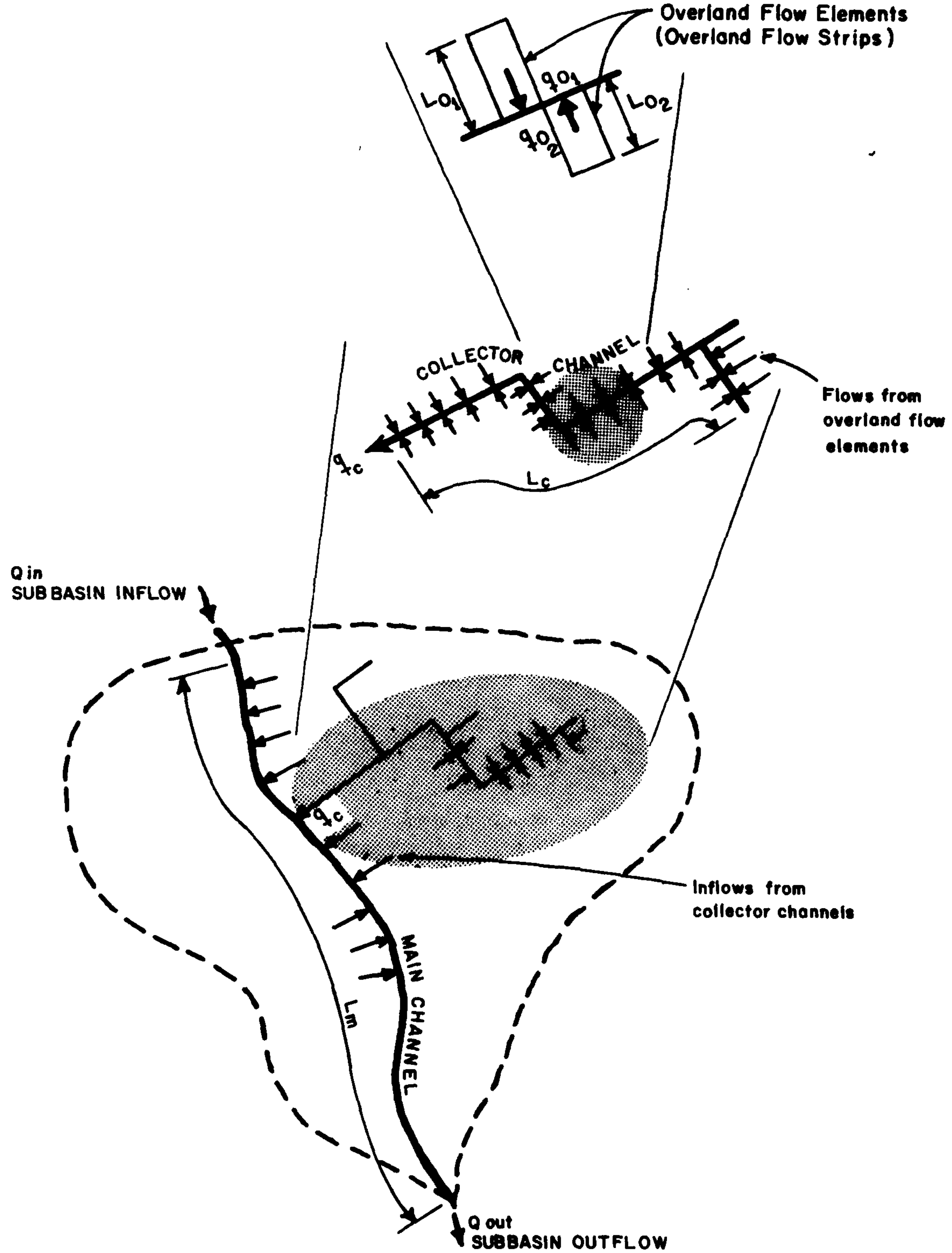

Figure 1.4. Relationships Between Flow Elements

is far more complicated than that of the simple model, the agreement between computed and actual discharge hydrographs is quite good.

To simulate the response of a complex watershed to precipitation from storm events a mathematical model made up of combinations of simple geometric components is constructed. Successful application of this approach begins with the description of unsteady, uniform flow over an idealized planar overland flow element of unit breadth (called an overland flow strip) for a number of given boundary conditions. A boundary condition represents known or assumed flow conditions that are specified by the user. Then, similar relationships for routing channel flows resulting from runoff from overland flow elements are developed. Once these relationships have been developed, combinations of simple elements can be made to describe basin and subbasin responses to storm events. Figure 1.4 summarizes the relationship between the three different types of elements.

Notice that overland flows are handled separately from the channel flow, because overland runoff demonstrates specific shallow flow properties which lead to a form of the equations of motion which differ from the form required for collector channel and main channel flows. These flows are calculated individually and then combined properly to preserve continuity and accuracy. This computational method has been shown to be quite efficient (Harley, et.al., 1972).

Figures 1.3 and 1.4 present a schematic representation of this approach. The governing equations used for this combined overland flow and river channel routing procedure are derived from the general St. Venant equations for unsteady gradually varied open channel flow.

For the overland flow portion of the model, shallow surface water runoff assumptions are applied to Equations (1.1) amd (1.2), resulting in a simple kinematic wave form of the equations. Similar simplifications are found to be valid and useful for the channel routing portions as well. Attempting the rigorous mathematical description of these complex phenomena (runoff and channel routing through the natural topography) would require exceedingly small spatial and temporal detail and result in a very large system of simultaneous equations. The kinematic wave form of the St. Venant equations provides a simplified description of the physical system in terms of surfaces and channels with homogeneous properties. The important concept of the overland portion of the model is that water is distributed over a wide area and at very shallow average depths until it reaches a well defined collector channel. In urban areas, two general types of overland flow surface are usually present: pervious and impervious. The mechanics of flow over both kinds of surfaces are similar; however, the slopes, flow lengths, roughnesses, and loss rates will differ. Roof tops, parking lots, and paved surfaces such as streets, are described as impervious areas. Lawns, fields, parks, etc., are pervious areas. The percentages of the total subbasin area that are impervious and pervious are stipulated by the user. The model develops the runoff flows from the rainfall intensity and loss rates specified for the pervious and impervious areas in the basin. After the overland runoff is routed down the length of the overland flow strip it is then distributed uniformly along the collector system which represents rivulets, channels

gutters, sewers, and storm drains such as shown in Figures 1.3 and 1.5. Once the runoff flows enter the collector system (see Figure 1.5) they move through it picking up additional (uniformly distributed) lateral inflow from adjacent runoff strips. These collector flows eventually reach a main channel where they are then routed as open channel stream flows through the system.

The following few paragraphs will review the basic concepts associated with the kinematic wave simplification of the momentum equation and present some criteria developed by Lighthill and Whitham (1955) that relate to its validity for various flow conditions. Following sections will then present the details of the three different segments of flow routing (overland, collector and stream routing) and how they can be applied to problems in urban hydrology.

The Kinematic Wave Form of the Momentum Equation is a Simple Stage-Discharge Relationship

Recall that kinematic waves occur when the dynamic terms in the momentum equations are negligible. This allows one to assume that the bed slope is approximately equal to the friction slope ($S_0 = S_f$). Under these conditions and if there is no appreciable backwater effect, the discharge can be described as function of depth of flow only, for all x and t.

$$Q = \alpha y^m \tag{1.3}$$

where, Q is discharge in cfs and α and m are kinematic wave routing parameters which are directly related to the basin and flow characteristics.

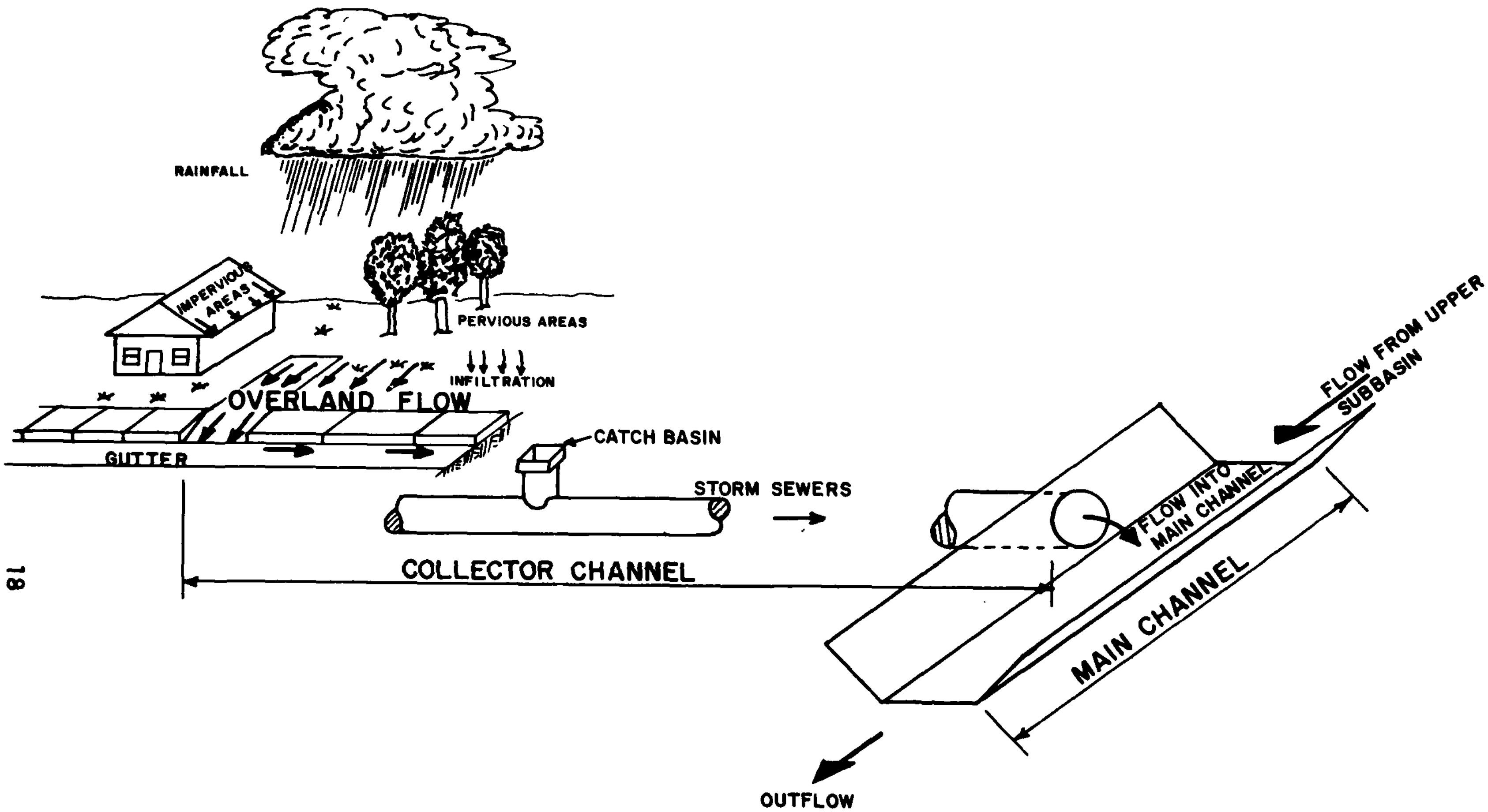

Figure 1.5 Typical Urban Drainage Pattern

This can be best appreciated using Henderson's (1966) approach of normalizing the momentum equation with a steady uniform discharge called Q_n. Rearrangement of the normalized form of Equation (1.2) yields

$$Q = Q_n \left[1 - \frac{1}{S_o}\left(\frac{\partial y}{\partial x} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{1}{g}\frac{\partial u}{\partial t} + \frac{q_L u}{gy}\right)\right]^{1/2} \qquad (1.4)$$

If the sum of the terms to the right of the minus sign if much less than one (i.e., pressure, inertia and local inflow are relatively small compared to S_o), then unsteady flows are nearly uniform and may be approximated by a series of normal flows, e.g.,

$$Q \simeq Q_n \qquad (1.5)$$

Normal flows of this nature can be described by a depth-discharge relationship, such as Equation (1.3) (Overton and Meadows, 1976). This describes kinematic flow and provides a simple method for calculating flows from stormwater runoff.

Lighthill and Whitham (1955) showed that for Froude numbers less and 2, the dynamic component decays exponentially and the kinematic wave ultimately dominates. As was mentioned in the introduction, this means that no visible surface wave is observed; only the rise and fall of the water surface can be seen (Figure 1.1). Woolhiser and Liggett (1967) studied the characteristics of a rising hydrograph for a large variety of flow conditions and found that the dynamic component in Equation (1.4) will be dampened enough to be neglected, provided that

$$k = \frac{S_o L}{y F^2} > 10 \qquad (1.6)$$

where L is the length of the plane, $\mathbf{F}$ is the Froude number $\mathbf{F} = V/\sqrt{gy}$, y is the mean depth, S_0 bed slope, and k is the dimensionless kinematic flow number. Practical evaluation of k is difficult because it may be hard to estimate L, y or $\mathbf{F}$ precisely for natural flow conditions without collecting field measurements. Results from Woolhiser and Liggett's (1967) study allows Equation (1.2) to be greatly simplified. These results are summarized in Figure 1.6 [where $\dot{Q}_*$ and t_* are dimensionless discharge $Q_* = (q_L L/Vg)$ and time $t_* = (tV/L)$ respectively]. For a value of k of 10, an approximate 10% error in the calculated discharge hydrograph would result by deleting the dynamic terms from the momentum equation. Notice that as k increases above 10, however, that the error in discharge decreases rapidly. A "true kinematic" solution results as k approaches infinity, but for engineering purposes flows characterized by k > 10 can be approximated reasonably well with the kinematic wave form of the momentum equation, e.g., Equation (1.3). Continuing research will attempt to develop practical methods of evaluating k and, therefore, the applicability of kinematic wave simplifications from typical basin data and flow conditions.

Derivation of the relationships used for overland flow, collectors and main channels will be presented next.

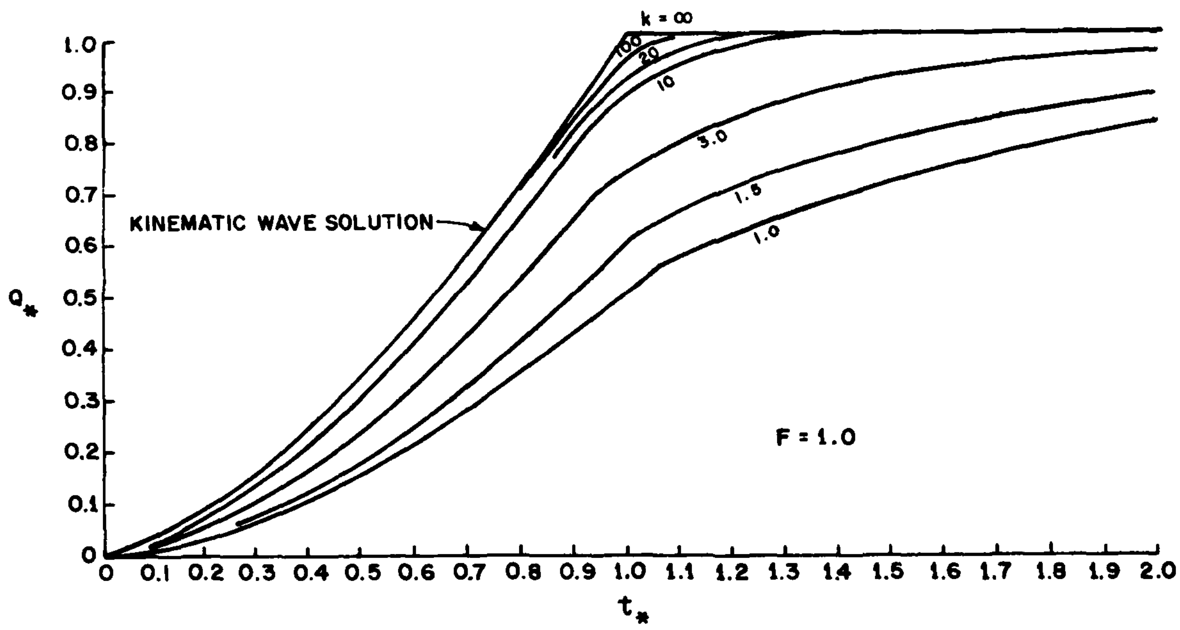

Figure 1.6 The Rising Hydrograph – Variation with k, for F = 1

(from Liggett and Woolhiser, 1967)

Development of α and m for Various Cross Section Shapes

The following sections will present example derivations of α's and m's for the different cross sectional shapes considered by the HEC-1 computer model.

Overland Flow Relationship

The Kinematic Wave equation for an overland flow segment on a wide plane with very shallow flows can be derived from Manning's equation and Equation (1.3). Consider the flow down an overland flow strip of unit breadth as shown in Figure (1.3). The steady discharge from a unit strip can be described with Manning's relationship.

$$Q = \frac{1.486}{n} R^{2/3} S_o^{1/2} A \tag{1.7}$$

where for very shallow flows that are at a depth of y_o, R (the hydraulic radius) and A are simply $(y_o \bullet 1)/1$ and $(y_o \bullet 1)$ respectively. Substitution of these values for R and A into Equation (1.7) and simplifying yields

$$Q = \frac{1.486}{N} S_o^{1/2} y_o^{5/3} \tag{1.8}$$

Notice also that Manning's n has been replaced with an appropriate N which describes the properties of the runoff surface being modeled. Values of N are usually greater than Manning's n. Table 1.1 presents suggested ranges of values for N for various surface conditions. These values were obtained from previous field and laboratory investigations for overland flow.

Because the discharge represented by Equation (1.8) is per unit breadth one can substitute the previously defined q for Q (see Equation (1.2) for units) to obtain discharge in terms of flow per unit breadth.

TABLE 1.1

EFFECTIVE ROUGHNESS PARAMETERS FOR OVERLAND FLOW

Surface	N
Dense Growth*	0.4 -0.5
Pasture*	0.3 -0.4
Lawns*	0.2 -0.3
Bluegrass Sod**	0.2 -0.5
Short Grass Prairie**	0.1 -0.2
Sparse Vegetation**	0.05-0.13
Bare Clay-Loam Soil (Eroded)**	0.01-0.03
Concrete/Asphalt - Very Shallow Depths* (depths less than 1/4 inch)	0.10-0.15
- Small Depths* (depths on the order of 1/4 inch to several inches)	0.05-0.10

* from Crawford and Linsley (1966)

** from Woolhiser (1975)

$$q = \left(\frac{1.486}{N} S_o^{1/2}\right) y_o^{5/3} \qquad (1.9)$$

Rewriting Equation (1.3) in terms of discharges per unit width where the subscripts "o" indicate variables associated with overland flows.

$$q_o = \alpha_o y_o^{m_o} \qquad (1.10)$$

where

$\alpha_o = \left(\frac{1.486}{N} S_o^{1/2}\right)$

$m_o = 5/3$

S_o = average slope of overland flow element

y_o = mean depth for overland flow

α_o = conveyance for particular runoff surface, slope, and roughness

Because there are two unknowns in Equation (1.10) another relationship is required for mathematical closure and a complete solution. A form of the continuity equation [see Equation (1.1)] provides the necessary second equation to complete the solution.

$$\frac{\partial y_o}{\partial t} + \frac{\partial q_o}{\partial x} = (i - f) \qquad (1.11)$$

where (i-f) is the rate of excess rainfall (rainfall-infiltration) in ft/sec, q_o is the discharge per unit width in cfs/ft, t is time in seconds, and y_o is the mean depth of overland flow in ft. Together Equations (1.10) and (1.11) form the complete kinematic wave equations for overland flow.

If Equation (1.10) is substituted into Equation (1.11) one obtains

$$\frac{\partial y_o}{\partial t} + \alpha_o m_o y_o^{(m_o - 1)} \frac{\partial y_o}{\partial x} = i - f \qquad (1.12)$$

which has only one dependent variable so that it can be solved to give a relationship for y_o in terms of x, t, and the excess rainfall intensity (i-f). Once y_o is found, it can be substituted back into Equation (1.10) to obtain a value for q_o. This procedure provides the necessary information to be able to determine the time dependent discharge from the overland flow elements.

Collector and Main Channel Routing Relationships

For the collector system (which represents rivulets, storm drains, and sewer pipes) and stream channel segments, simple cross section shapes have been used to simulate prototype channels. It has been found that appropriate usage of simple triangular, trapezoidal, and circular channel shapes can provide an accurate representation of the response of the prototype.

Flows entering the collectors and the stream channels can consist of both flows from upstream sections and lateral inflows from adjacent catchment surfaces. These representative channels are described by their slope, length, cross sectional dimensions, shape, and Manning's n value. The standard Manning's n is used here because collector and stream flows behave more as normal open channel flows. The basic form of the equations for kinematic wave routing of collector and streamflows are similar to those developed for shallow overland flow [Equations (1.10)

and (1.11)]. The kinematic wave equations for collector and stream flow routing are:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x} = q_o \tag{1.13}$$

$$Q_c = \alpha_c \, A_c^{m_c} \tag{1.14}$$

where

A_c	= cross sectional area of flow in ft^2
Q_c	= discharge in cfs
q_o	= lateral inflow per unit length in cfs/ft from Overland Flow Strips
t	= time in seconds
x	= distance along the stream in feet
α_c, m_c	= kinematic wave parameters for a particular cross sectional shape, slope and roughness

The reader should note that the subscripted variables used above are for a typical collector channel and are indicated as such with the subscript c. Identical relationships would be used for routing in the main channel but one may wish to identify them separately with a subscript such as m to indicate main channel (refer to "Definition of Terms").

Determination of α_c and m_c for Collectors and Streams

Values of α_c and m_c will be different for each differently-shaped cross section and will vary with effective Manning's n and channel slope as well. The basic channel shapes considered by the HEC-1 model are the

trapezoid and circle. Variation of the side slope and bottom width for the trapezoidal section allows one to develop rectangular and triangular channel shapes as well. These shapes are presented in Figure 1.7.

As was done for the development of the overland flow parameters α_o and m_o, one needs to define the proper values for α_c and m_c for the stream and collector system. Rather than derive all the α_c and m_c values for each differently shaped section, an easy-to-follow development for a simple triangular section will be presented as an example. The results for the remaining shapes will merely be presented because their derivations follow the same procedures.

Triangular Sections

Derivation of α_c and m_c for a triangular channel shape begins with the description of R_c and A_c in Equation (1.7) for a triangular shaped cross section. The reader should refer to Figure 1.7 and to the definition of terms after Equations (1.13) and (1.14).

$R_c = A_c/P_c$ = (Area of triangular cross section/ Wetted Perimeter) = Hydraulic Radius

where

$P_c = 2\left(\sqrt{1+z^2}\right) \cdot y_c$ = Wetted Perimeter

$A_c = 1/2\,(2y_c \cdot z) \cdot y_c = zy_c^{\,2}$ = cross sectional area

z = side slope ratio

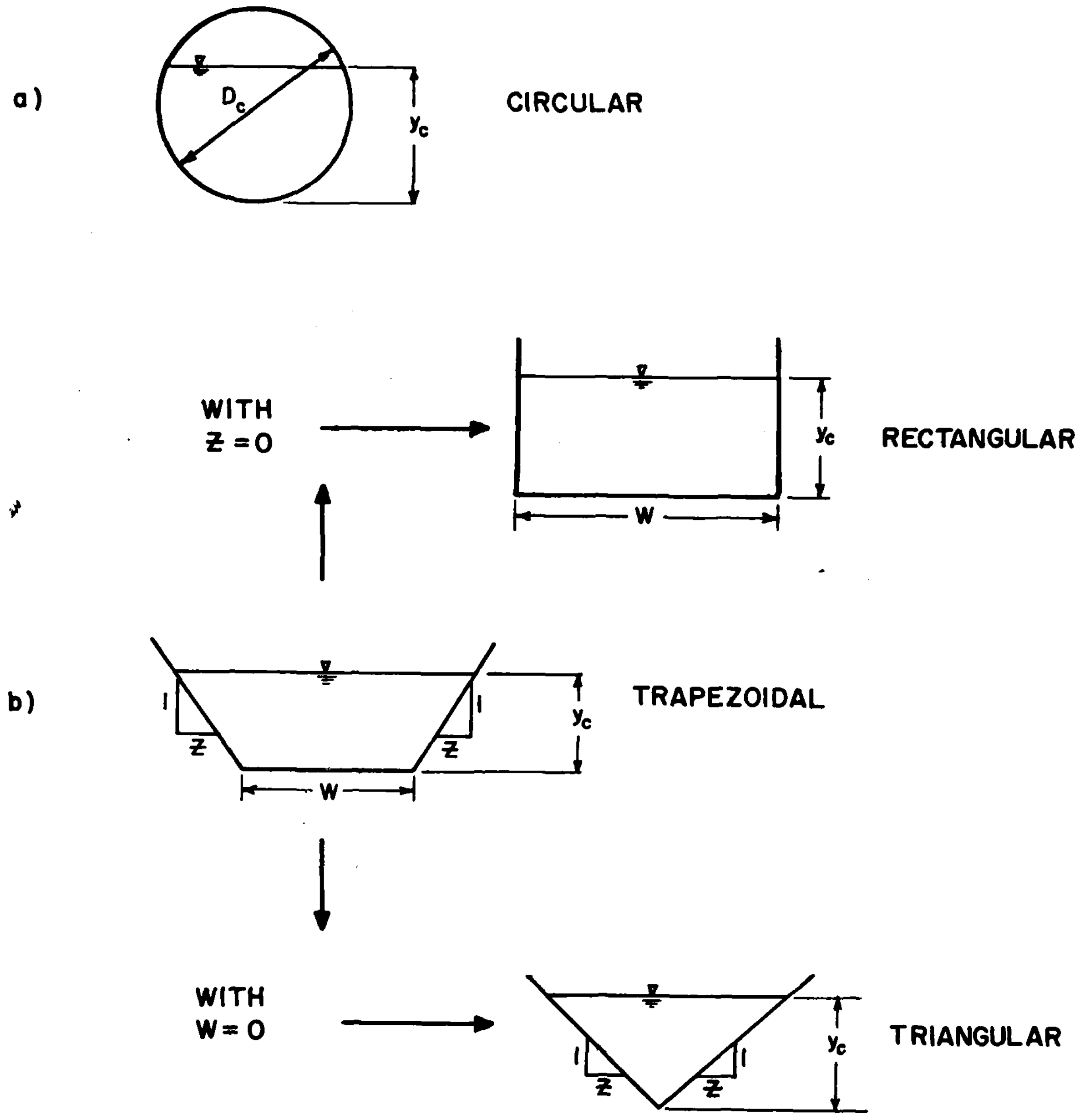

Figure 1.7 Two Basic Channel Shapes and their Variations Used by the HEC-1 Flood Hydrograph Package for Kinematic Wave Stream Routing

Now the values of A_c and R_c may be substituted into Equation (1.7).

$$Q_c = \frac{1.486}{n} S_c^{1/2} \frac{A_c^{2/3}}{P_c^{2/3}} A_c = \frac{1.486\ S_c^{1/2}}{n} \frac{A_c^{5/3}}{P_c^{2/3}} \tag{1.15}$$

Defining Φ as $(1.486\ S_c^{1/2}/n)$, and substituting the appropriate value for A_c and P_c into Equation (1.14) and simplifying, gives

$$Q_c = \Phi \frac{z^{5/3}\ y_c^{10/3}}{(2^{2/3})(1+z^2)^{1/3}\ y_c^{2/3}} = \Phi \frac{z^{1/3}\ (z^{4/3}\ y_c^{8/3})}{(2)^{2/3}\ (1+z^2)^{1/3}} \tag{1.16}$$

$$= \frac{1.486}{1.587} \frac{S_c^{1/2}}{n} \left(\frac{z}{1+z^2}\right)^{1/3} (z \cdot y_c^2)^{4/3} \tag{1.17}$$

$$\text{or } Q_c = \frac{0.94\ S_c^{1/2}}{n} \left(\frac{z}{1+z^2}\right)^{1/3} A_c^{4/3} \tag{1.18}$$

Equation (1.18) may now be written in the form of Equation (1.3)

$$Q_c = \alpha_c\ A_c^{m_c} \tag{1.19}$$

where

$$\alpha_c = \frac{0.94\ S_c^{1/2}}{n} \left(\frac{z}{1+z^2}\right)^{1/3}$$

and

$$m_c = 4/3$$

Rectangular Sections

A rectangular shape is obtained by stipulating that z in Figure 1.7-b is zero. This produces a channel w feet wide with vertical walls. Man-made channels and rectangular concrete drain sections may be represented by this shape. Following similar procedures which produced Equations (1.18) and (1.19), two separate relationships for rectangular shapes are easily derived. One for a very wide rectangular channel where w is much greater than the depth y_c, and the other for a rectangular channel that may have comparable depths and breadths, e.g., $w \approx y_c$.

For the wide shallow channel case:

$$Q_c = \alpha_c A_c^{m_c} \tag{1.20}$$

where

$$\alpha_c = \frac{1.486\, S_c^{1/2}}{n} \cdot w^{-2/3}$$

and

$$m_c = 5/3$$

For the rectangular channel where $w \approx y_c$:

$$Q_c = \alpha_c A_c^{m_c} \tag{1.21}$$

where

$$\alpha_c = \frac{0.72\, S_c^{1/2}}{n}$$

$$m_c = 4/3$$

Trapezoidal Sections

The trapezoidal section is one of the two basic sections considered by the general HEC-1 code. Modifications of the trapezoidal shape produced the rectangular and triangular alternatives defined above. When describing a trapezoidal section it is important to define the side slopes z and the channel bottom widths w accurately (see Figure 1.7). It is not possible to derive a simple relationship for α_c and m_c from the geometric properties alone, so it becomes necessary to fit α_c and m_c to the Manning equation at two or more depths y_c and use numerical techniques of fitting the kinematic equation to these values to obtain values for α_c and m_c for various flow conditions.

Kinematic wave equation $\quad Q_c = \alpha_c A_c^{m_c}$

Manning equation for a trapezoid

$$Q_c = \frac{1.486\ S_c^{1/2}}{n} (A_c')^{5/3} \left[\frac{1}{w + zy_c(1+z^2)^{1/2}}\right]^{2/3} \qquad (1.22)$$

where A_c' = the area of the effective cross section at depth y_c

Values of m_c will vary from 4/3 for a triangular section to 5/3 for a wide rectangular section.

Circular Sections

Circular sections can be used to model storm or sewer pipes in urban areas. Resource Analysis, Inc. (1975) derived the following relationships for α_c and m_c for typical circular sections such as that

shown in Figure 1.7-a which apply to pipe sections flowing less than 90% full.

$$Q_c = \alpha_c A_c^{m_c} \tag{1.23}$$

where

$$\alpha_c = \frac{0.804 S_c^{1/2}}{n} D_c^{1/6}$$

and

$$m_c = 1.25$$

and

D_c = the diameter of the circular section in feet

Numerical Solution of the Kinematic Wave Equations

The HEC-1 Flood Hydrograph Package solves the kinematic wave equations using finite difference numerical techniques. Harley, (1975); Resource Analysis, Inc., (1975); and Bras, (1973) present the details of these methods which have been perfected after years of development and testing. As with standard numerical procedures, time is discre*ized in constant steps of Δt and distance in steps of Δx. The rainfall excess (i-f) is assumed constant within each time step Δt, but does change from time step to time step, to simulate the variability occurring within a storm event.

Recall that Equations (1.13) and (1.14) were the kinematic wave equations for collector and stream flow routing. If Equation (1.14) is substituted into Equation (1.13), the following relationship which has A_c as the only dependent variable will be obtained

$$\frac{\partial A_c}{\partial t} + \alpha_c m_c A_c^{(m_c-1)} \frac{\partial A_c}{\partial x} = q_o \qquad (1.24)$$

Numerical solution of Equation (1.24) produces a relationship for A_c in terms of x, t, and q_o. These values of A_c may then be substituted into Equation (1.14) to solve for Q_c. This simple procedure (described previously for overland flows) allows the calculation of Q_c as a function of the segment length L_c and time t. Therefore, one can describe the discharge hydrograph from each of the segments that are of length L_c. If these discharges, Q_c, represented discharges from local collector channels, then HEC-1 will distribute them uniformly as lateral inflow into the main channel or stream as shown in Figures 1.3 and 1.4. Calculation of the resulting discharge hydrograph from the main channel or stream is then calculated in an identical manner as was just presented. The equations used would be identical to Equations (1.23) and (1.24) except the subscript c would be replaced with an m everywhere throughout Equations (1.23) and (1.24). This provides a simple straight forward procedure for calculating first the overland flows, then the flows through the collector channel system and finally the discharges in the main stream.

Detailed development of the specific finite difference equations, the coding procedures and boundary requirements can be found in the following references: (Lighthill and Whitham, 1955; Bras, 1973; Resource Analysis, Inc., 1975; Hydrologic Engineering Center, 1979). The coding modifications that were made to include the kinematic wave routing procedure in HEC-1 was done by Resource Analysis, Inc., under

the direction of Dr. B. Harley. The algorithms used are based on those developed for the MITCAT Catchment Simulation Model (Resource Analysis, Inc., 1975). The following section presents a brief review of the numerical methods and procedures utilized by the HEC-1 program to perform kinematic wave routing.

Finite Difference Solutions of the Kinematic Wave Equations

The movement of a flood wave down a stream or along an overland flow surface can be followed by monitoring the times and locations where specific water surface elevations and discharges occur. For example, if lateral inflow is assumed to be zero for a moment, a flood wave can be followed in time by noting the times when a flow of a given magnitude, or a stage of a given magnitude occurs at successive downstream stations along the channel. Flood routing methods, such as the kinematic wave method used in HEC-1, depend upon certain numerical techniques to solve the governing equations which describe the movement, stage and discharge characteristics of a flood wave as it propagates downstream. Overton and Meadows (1976) and Mahmood and Yevjevich (1975) present discussions of several of the different methods currently in use for hydrologic engineering studies. The numerical method currently employed by the HEC-1 program to solve the governing equations (e.g., Equations (1.10) and (1.11) or (1.14) and (1.24) is a finite difference method (Hydrologic Engineering Center, 1979). A finite difference method (FDM) presents a "pointwise approximation" to the governing partial differential equations. The FDM uses simple difference equations which replace the partial differential equations for an array of stationary grid points located

in the space-time (x-t) plane (see Figure 1.8). The intersections of the lines in Figure 1.8 define the time and space points at which the discharge and water surface elevation are computed. Lines parallel to the x-axis are called time lines, and lines which are parallel to the t-axis are space lines. The regular pattern formed by the intersections of time and space lines is called the "computational network." Those points, as noted on the figure, that are marked by solid dots represent computation points (called nodes). Solutions to the governing equations via the FDM will be computed at each of these nodes. Computations advance along the downstream direction for each time step Δt until all the flows and stages are calculated along the entire distance L. Then the computation is advanced ahead in time by one Δt and the computations for discharge and water surface elevation are performed once again.

Also shown on Figure 1.8 are the solution curves (the dashed lines) that represent solutions obtained from another method called the "method of characteristics." The method of characteristics is not an available alternate solution technique in HEC-1. As can be seen in Figure 1.8, the solution curves (called characteristic curves) do not always intersect at a node where evenly spaced time and space lines intersect. Because of this, additional interpolation would be required to obtain solutions at evenly spaced nodes. To avoid this interpolation, the method of characteristics is not used in HEC-1. It is mentioned here though, because the characteristic curves represent locations in the x-t plane where specific flow properties, such as wave celerity c,

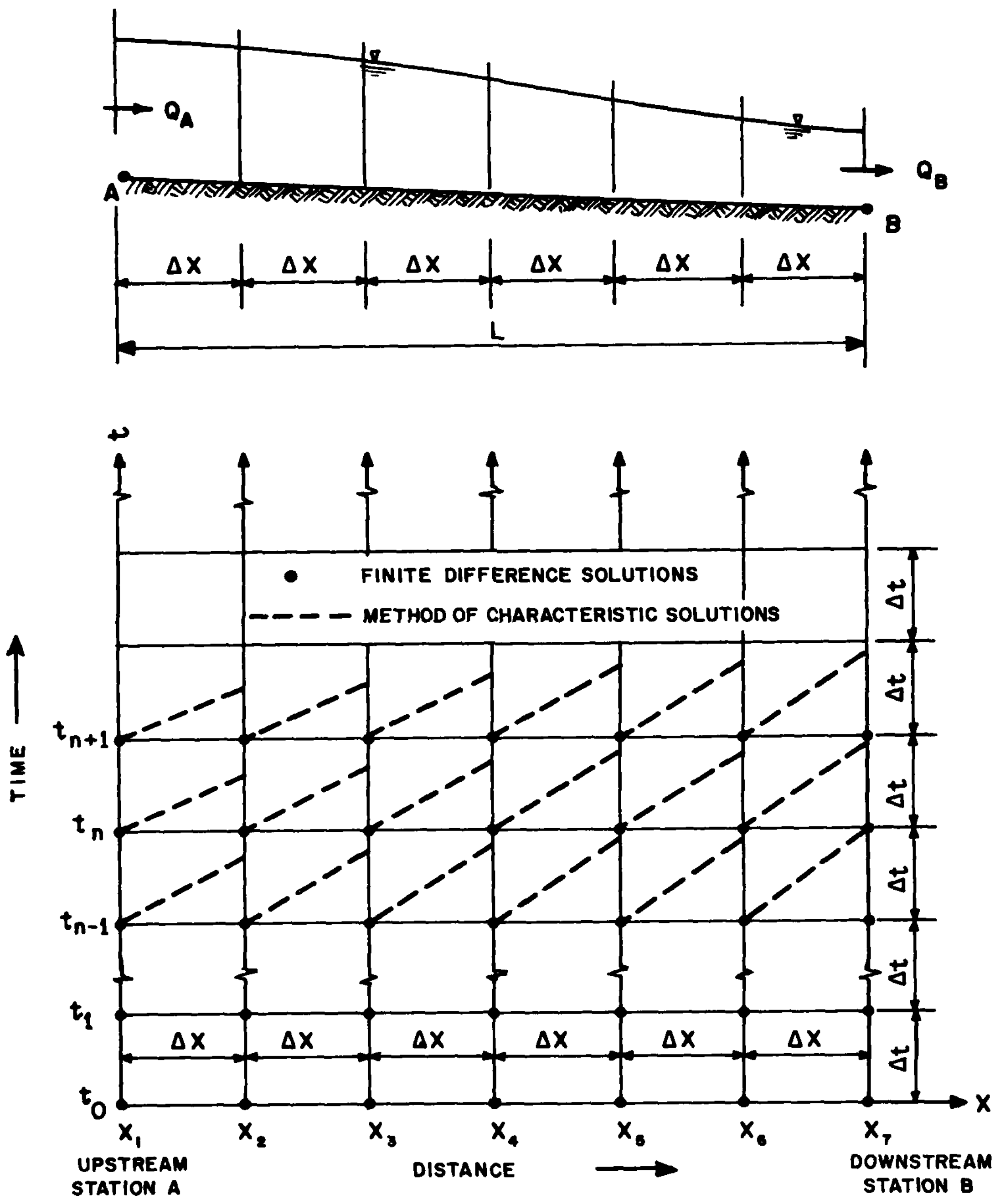

Figure 1.8 Characteristic Curves on a Fixed $\Delta X - \Delta t$ Grid

remain constant for each time step Δt. The importance of this is shown below when the specific numerical solution methods (the standard form and conservation form of the governing equations) are explained.

Detailed methods for the approximation of derivates by finite differences can be found in Carnahan, Luther and Wilkes (1969). By simply combining the appropriate finite difference approximations for first- and second-order derivates, complete partial differential equations, such as Equations (1.14) and (1.24) can be recast in terms of finite differences instead of partial derivatives. These new equations are approximations of the original equations but are now in a form which can be easily handled numerically, especially with the aid of high-speed computers. As an example of the procedure; the first-order partial derivative $\partial Q/\partial x$ is approximated using a backward finite difference method below:

$$\frac{\partial Q}{\partial x} \approx \frac{\Delta Q}{\Delta x} \approx \frac{Q_{i,j} - Q_{i-1,j}}{\Delta x} \tag{1.25}$$

The following sections will discuss how similar finite-difference approximations are applied to solve the governing kinematic wave equations in space and time.

The governing equations developed in the previous sections consisted of a pair of equations for each of the different kinds of flow elements; e.g., Equations (1.10) and (1.12) for overland flow elements, Equations (1.13) and (1.14) for collector elements, and two additional equations identical to (1.13) and (1.14) (only with different subscripts) for the main channel routing elements. Rather than treat each of these three

pairs of equations separately, the solution details will be developed for one set only because they are basically all the same. The HEC-1 program handles each of these three different kinds of flow elements by using the following computational sequence: computations start with the determination of overland flows which are then input as uniformly distributed lateral inflows into the collector elements which, in turn, modify the flows and distribute these collector flows uniformly and laterally along the main channel. The main channel routes the final flood wave through the subbasin. A combination of several subbasins thus allows for the complete description of an entire basin during a storm event.

Standard Form of the Kinematic Wave Equations

Consider Equation (1.13) which relates flow Q_c within a collector element to the collector channel cross sectional area A_c and Equation (1.14) which is the continuity equation:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x} = q_o \qquad (1.13)$$

$$Q_c = \alpha_c A_c^{m_c} \qquad (1.14)$$

It is assumed that the kinematic wave coefficients α_c and m_c are constant for any given system of channel elements. Differentiation of the flow Equation (1.14) with respect to x and substitution of this into Equation (1.13) gives:

$$\frac{\partial A_c}{\partial t} + \alpha_c m_c A_c^{(m_c - 1)} \frac{\partial A_c}{\partial x} = q_o \qquad (1.27)$$

Notice that Equation (1.27) is the same as the previously derived Equation (1.24). Referring to Figure 1.9, the area A_c is known at points on the space-time grid for times prior to the current time (designated by the index j) and at points in space prior to the current location (indicated by the index i). Therefore the index (i,j) corresponds to the current time and space coordinates. Future times and space locations that are advanced by one Δt and Δx are indicated as j+1 and i+1, respectively. Similarly, one previous time and space location would correspond to a point on the space-time grid indexed by (i-1, j-1). In this way, each point (node) on the space-time grid can be indicated by a double subscript of i's and j's. This allows one to rewrite partial differential equations in terms of finite difference approximations of known quantities (located at previous times and space points) and unknown quantities (current time and space points). Therefore, the area A_c at point B in Figure 1.9 would be designated as $A_{c_{(i-1,j)}}$, at point C, $A_{c_{(i-1,j-1)}}$, at point D, $A_{c_{(i,j-1)}}$ and so forth. With this background, one can now express the governing equations in terms of finite differences using the previously defined indexing scheme and solve for the values of A_c and Q_c at the "current point A" in Figure 1.9.

$$\alpha m\, A^{(m-1)} \frac{\partial A}{\partial x} \simeq \alpha m\, [\bar{A}^{(m-1)}] \frac{\Delta A}{\Delta x}$$

$$= \alpha m \left[\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2}\right]^{(m-1)} \cdot \left[\frac{A_{(i,j-1)} - A_{(i-1,j-1)}}{\Delta x}\right] \quad (1.28)$$

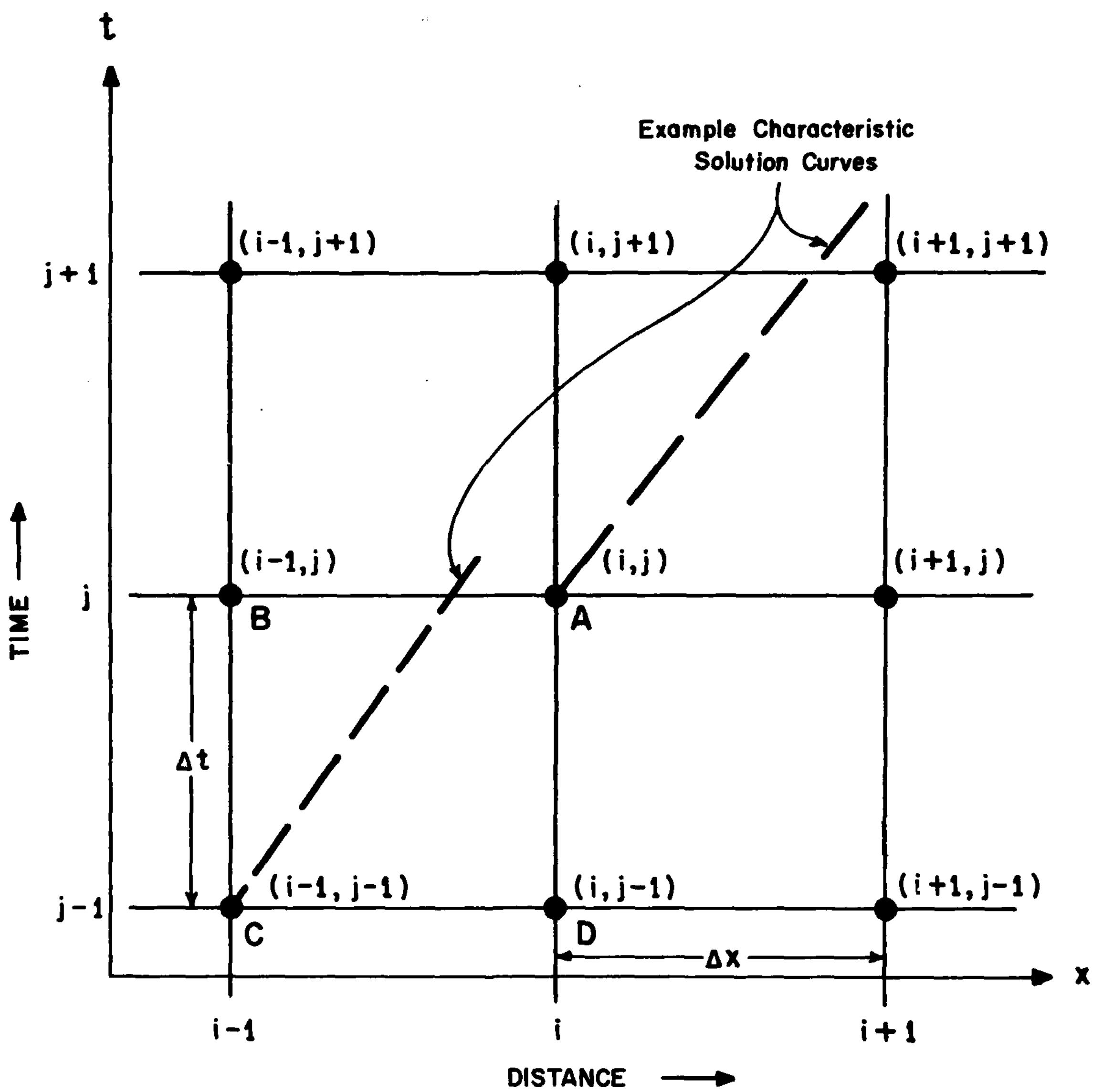

Figure 1.9 Space-Time Grid Used for Finite Difference Method

where the differential of the area A in the x-direction is taken as the difference between the values known at points C and D in Figure 1.9. Also, the area term which is raised to the (m-1) power in Equation (1.27) is considered to be an average area between points C and D in Figure 1.9.

Consider the time derivative term $\partial A/\partial t$ in Equation (1.27) next. It is evaluated between points A and D (see Figure 1.9), thus:

$$\frac{\partial A}{\partial t} \approx \frac{\Delta A}{\Delta t} \approx \frac{A_{(i,j)} - A_{(i,j-1)}}{\Delta t} \quad (1.29)$$

The lateral inflow term q is handled as an average lateral inflow which occurs within a time step t and is defined as $\overline{q}$ here to simplify the final form of the equation.

$$q \approx \frac{q_{(i,j)} + q_{(i,j-1)}}{2} = \overline{q} \quad (1.30)$$

Combining Equations (1.28), (1.29) and (1.30) produces the complete finite difference form of the original partial differential Equation (1.27):

$$\frac{A_{(i,j)} - A_{(i,j-1)}}{\Delta t} + \alpha m \left[\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2}\right]^{(m-1)} \bullet$$
$$\left[\frac{A_{(i,j-1)} - A_{(i-1,j-1)}}{\Delta x}\right] = \overline{q} \quad (1.31)$$

$A_{i,j}$ is the only unknown quantity in Equation (1.31) which can, therefore, be isolated and directly solved for.

$$A_{(i,j)} = \bar{q}\Delta t + A_{(i,j-1)} - \alpha m \frac{\Delta t}{\Delta x} \left[\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2}\right]^{m-1}$$

$$[A_{(i,j-1)} - A_{(i-1,j-1)}] \qquad (1.32)$$

Once $A_{i,j}$ is known, the corresponding flow $Q_{i,j}$ can be computed from Equation (1.14)

$$Q_{i,j} = \alpha(A_{i,j})^m \qquad (1.14)$$

Knowing $A_{i,j}$ and the cross sectional properties of the channel at location i, one can also compute the water surface elevation for that flow, time and location.

This provides a straight forward method of computing time varying discharges and water surface elevations along the channel.

Conservation Form of the Equation

The previous "Standard Form" of the equations applies in most situations where the average wave celerity $\bar{c}$ is less than the ratio of the computational space to time step, e.g., $\bar{c} < (\Delta x/\Delta t)$. When $\bar{c}$ is less than $\Delta x/\Delta t$ it is felt that the previous procedure will provide an accurate approximation for the kinematic wave characteristics of a flood wave. However, if $\bar{c}$ is greater than $(\Delta x/\Delta t)$ is is possible for flood wave characteristics to propagate more rapidly through space and time than the numerical approximation method can account for them. (This numerical stability criteria can be associated with the familiar Courant conditon for stability of explicit finite difference schemes.) For this reason, an alternate form of the approximate finite difference equations is needed.

The "Conservation Form" of the governing equations is therefore, applied when the average celerity $\bar{c}$ of the flood wave is greater than the ratio $\Delta x/\Delta t$. In this form, the temporal derivatives are evaluated between points B and C in Figure 1.9 rather than A and D, while the spacial derivatives are evaluated between points B and A rather than D and C. In this way rapidly advancing flood wave characteristics, such as the example characteristic curves indicated in Figure 1.9, can be more accurately accounted for. HEC-1 checks to see if $\bar{c}$ is less than $\Delta x/\Delta t$ for each time step. If it is, the Standard Form of the equations are used; if not, the Conservation Form of the equations are used.

The conservation form of the spacial derivative of discharge will be evaluated between points B and A in Figure 1.9.

$$\frac{\partial Q}{\partial x} \simeq \frac{\Delta Q}{\Delta x} = \frac{Q_{(i,j)} - Q_{(i-1,j)}}{\Delta x} \qquad (1.33)$$

and the temporal derivative of area will be evaluated between points B and C

$$\frac{\partial A}{\partial t} \simeq \frac{\Delta A}{\Delta t} = \frac{A_{(i-1,j)} - A_{(i-1,j-1)}}{t} \qquad (1.34)$$

Substitution of new "Conservation Form derivatives above into the continuity equation produces

$$\frac{Q_{(i,j)} - Q_{(i-1,j)}}{\Delta x} + \frac{A_{(i-1,j)} - A_{(i-1,j-1)}}{\Delta t} = \bar{q} \qquad (1.35)$$

Solving for the only unknown, $Q_{i,j}$ gives

$$Q_{i,j} = Q_{(i-1,j)} + \bar{q}\,\Delta x + \frac{\Delta x}{\Delta t}\left[A_{(i-1,j)} - A_{(i-1,j-1)}\right] \qquad (1.36)$$

Knowing $Q_{i,j}$, allows the area at the point i,j to be determined from the flow equation:

$$A_{i,j} = (Q_{i,j}/\alpha)^{1/m} \tag{1.37}$$

This concludes the introductory development of the general kinematic flow equations and a brief discussion of their numerical solution techniques. The following chapter will present additional background information and the details necessary for the effective application of these methods to solve problems in urban hydrology.

ACKNOWLEDGMENTS

The basic approach for the subdivision of the basin into the various elements listed here was developed and coded through discussions with John Peters, Art Pabst and Paul Ely of the Hydrologic Engineering Center. Wayne Pearson assisted in the calculations used in the examples found in Chapter 2.

The development and programming of the kinematic wave routing routines in computer program HEC-1 was done under contract to the Hydrologic Engineering Center by Resource Analysis, Incorporated, under the direction of Dr. Brendan Harley.

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CHAPTER 2

APPLICATION OF KINEMATIC WAVE ROUTING TECHNIQUES USING HEC-1

by J. J. DeVries[1]

INTRODUCTION

Computer program HEC-1 contains options for using kinematic wave theory to compute subbasin outflow hydrographs and to route hydrographs through a stream reach (Hydrologic Engineering Center, 1979). These options provide an alternative to the unit hydrograph method for determining direct runoff. They also provide a streamflow routing technique which can be used in place of the Muskingum, modified Puls, and other methods available in HEC-1 (Hydrologic Engineering Center, 1973). These newly added features have had only limited use to date at the Hydrologic Engineering Center (HEC), although the general method has been used with success in a number of other hydrologic models. As with any type of hydrologic model, however, it is imperative that the modeler check the performance of his modeling effort against observed data. Use of the model without a procedure for verifying its ability to correctly simulate the behavoir of a given basin is strongly discouraged.

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The purpose of this chapter is to describe how the kinematic wave method in HEC-1 may be applied. A discussion of kinematic wave theory as it relates to this program is given in Chapter 1 of this document. The papers by Lighthill and Whitham (1955), Harley, et.al. (1972), and Woolhiser (1975) are recommended for general background on the kinematic wave method.

One of the attractive features of the kinematic wave approach to rainfall-runoff modeling is that the various physical processes of the movement of water over the basin surface, with the attendant infiltration, flow into stream channels, and flow through the channel network are considered. Parameters, such as roughness, slope, catchment lengths and areas, and stream channel dimensions are used to define the processes.

The various features of the irregular surface geometry of the basin are generally approximated by either of two types of basic elements: (1) an overland flow element, and (2) a stream or channel flow element. In the modeling process described here, one or two overland flow elements (designated as overland flow strips) are combined with one or two channel flow elements to represent a subbasin. An entire basin is modeled by linking the various subbasins together.

Because the descriptions of the various elements comprising the model are directly related to physical parameters, the model can be easily modified, and changes which represent changes in land use in the basin can be made using parameters which describe these new uses. This makes kinematic-wave-type models very useful for urban studies

because the effects of increasing urbanization can be accounted for by changing the parameters describing the basin.

The various topics which are covered in this chapter include basin modeling procedures, a description of the elements used in kinematic wave calculations, and procedures for selecting the parameters. An example problem is presented to illustrate HEC-1 input and ouput data, and effects of changes to numerical values of the parameters are discussed. Results of a "hand" calculation are given in Appendix A to illustrate the basic solution procedure.

BASIN MODELING

The modeling process starts with a description of the topologic structure of the basin: drainage basin boundaries, stream and drainage channels, and the logical relationships between the drainage areas and the channels. The definition of the drainage boundary will depend on the objective of the study being conducted, as well as the topological character of the basin. Studies dealing with urban hydrology usually require delineation of subbasins that are smaller than 2 mi^2 in extent (about 5 km^2). Studies dealing with the effects of channel modifications may permit use of large areas; however, as the area is increased the assumptions required to apply the kinematic wave method become more tenuous. In general, subbasin areas should be limited to a maximum of four to five square miles.

A typical urban drainage system is shown in Figure 2.1. Rain falls on two general types of surfaces: (1) those that are essentially impervious, such as roofs, driveways, parking lots and other paved areas;

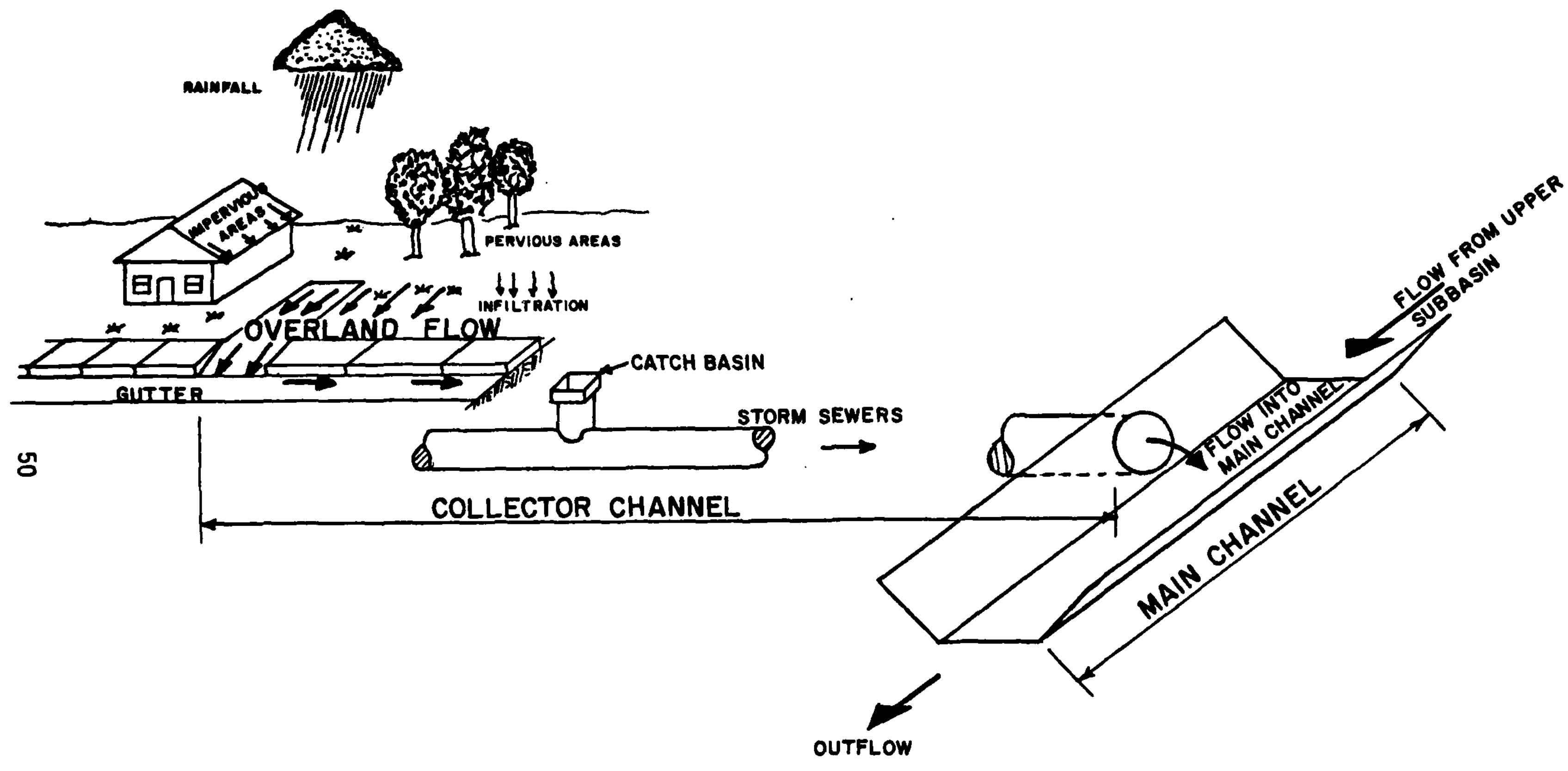


Figure 2.1 Typical Urban Drainage Pattern

and (2) pervious areas, most of which are covered with vegetation and have numerous small depressions which produce local storage of rainfall. It is assumed in the model that water initially travels over these surfaces as sheet flow; however, in a relatively short distance the water begins to collect in small streams or rivulets and the process of stream or channel flow begins. For impervious areas, the distance to the first channel (say a gutter) is typically thirty to one hundred feet, while for pervious surfaces the longest distance a drop of water must travel to reach a channel is on the order of one hundred to several hundred feet.

Water collected by the street gutters, travels no more than a few hundred feet until it enters catch basins which are connected to sewers. These sewers are typically 1.5 to 2 feet in diameter for the local drains. The local drains are connected in turn to larger and larger drains which feed the main storm drain. In many areas the main storm drains are open channels or streams. In major urban areas the main storm drains are often large closed-conduit sections, but these storm drains are usually designed to flow only partially full, and therefore, the kinematic wave routing approach (which assumes open channel flow) is appropriate.

There are certain weaknesses inherent in the kinematic wave routing approach which should be kept in mind by the modeler. These include the following: (1) in kinematic wave routing, the theory does not provide for attenuation of the flood wave. As a consequence, peak flows may be over estimated. (2) Surcharging of storm drains frequently occurs during major storm events; no explicit provision for surcharging is provided

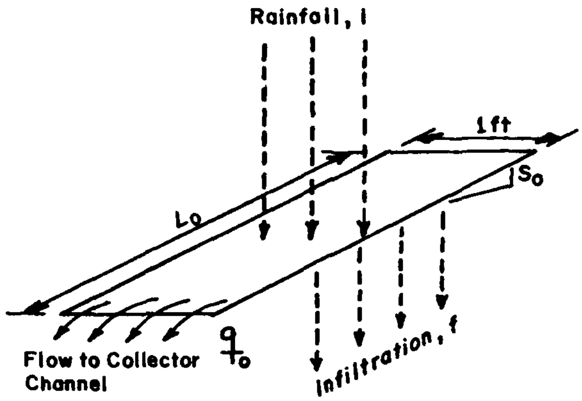
by the method, however. (3) Also, ponding and local storage of water during major events is not accounted for. This might include overbank storage, ponding in the streets, etc., which occur during the large storms which are of primary interest and may not occur to as great a degree in the events available for calibration and verification of the model. The modeler, therefore, should analyze the program results to see if this is happening.

ELEMENTS USED IN KINEMATIC WAVE CALCULATIONS

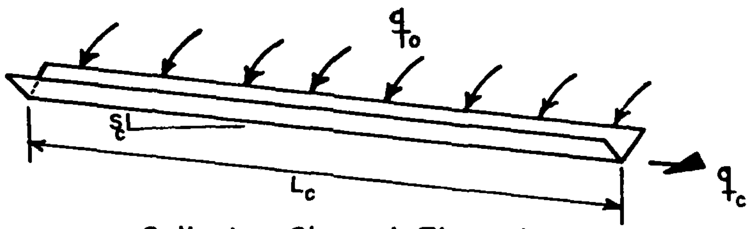
The runoff process described above is idealized in HEC-1 through the use of the following flow elements: (1) one or two typical overland flow elements, (2) a typical collector channel element, and (3) a main channel element. These generally provide the necessary detail for modeling the runoff process in urban basins. Schematic drawings of these elements are shown in Figure 2.2. Figure 2.3 illustrates the realtionships of the various types of elements to each other. The elements are specified to represent typical features of the basin, and thus the parameters chosen for the individual elements should be representative of the entire subbasin. The runoff simulation process is automatically expanded from the typical elements to the whole subbasin by the program. Because land use and development practices are usually very similar within a selected hydrologic unit, assigning a single value to a given parameter usually gives good results.

Overland Flow Elements

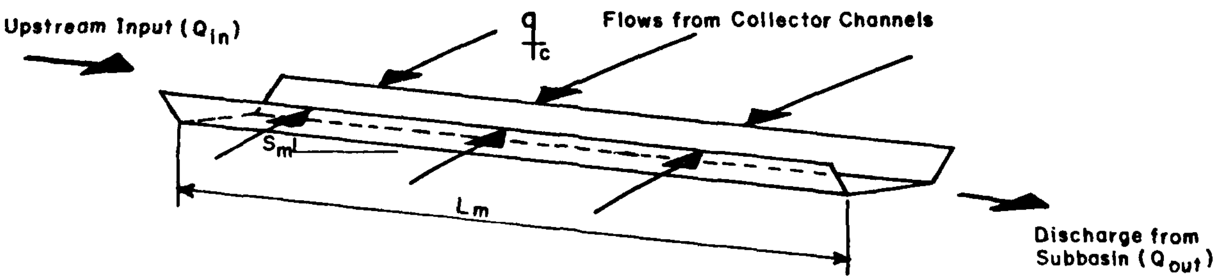
The basic overland flow element is simply a sloping rectangular plane surface upon which the rain falls. In the computer program it is treated as a strip of unit width (one foot or one meter wide). Some



Overland Flow Element



Collector Channel Element



Main Channel Element

Figure 2.2 Elements Used in Kinematic Wave Calculations

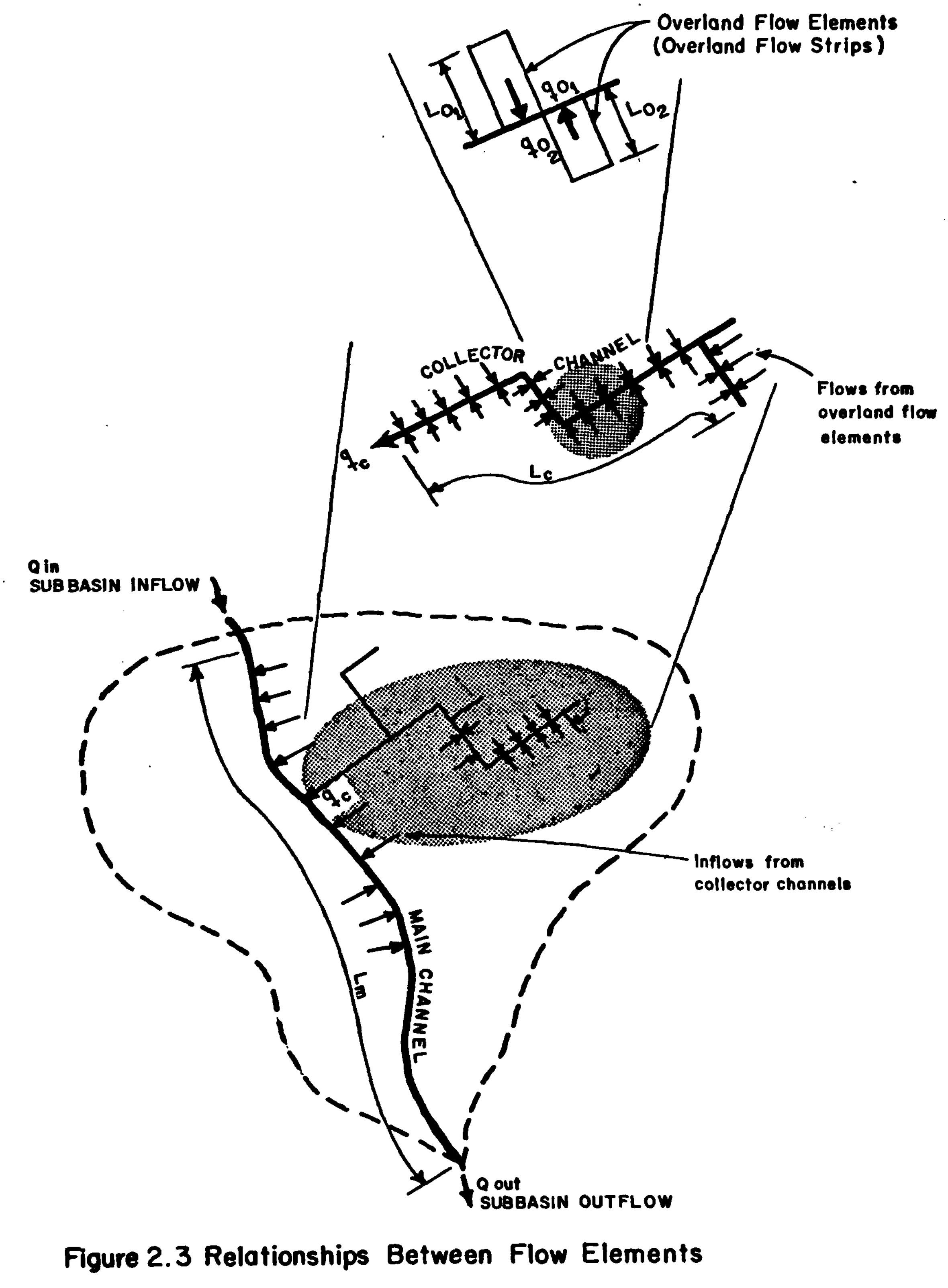


Figure 2.3 Relationships Between Flow Elements

of the rainfall is lost by infiltration; the remainder runs off the lower edge of the plane into a channel. Infiltration losses may vary with time or be constant, and a different loss rate can be specified for each flow strip. The fraction of the element that is impervious can also be specified.

The basic kinematic-wave-analysis concept used in HEC-1 allows the use of either one or two overland flow surfaces, each discharging into a collector channel. For example, one element could represent all areas that are essentially impervious, with short lengths of flow (L_0) to the point where the flow becomes channel flow. Thus the element would represent driveways, roofs, street surfaces, etc.

The other overland flow element could then represent areas that are pervious and have higher resistance to flow, such as lawns, fields, and wooded areas. In general, the catchment flow lengths and roughness coefficients will be much greater for these areas. Again, the value of L_0 to be used is the representative maximum distance for water to travel as overland flow for this type of land surface.

The user of this method should think of the overland flow strips as representing typical flow surfaces rather than actual planar surfaces, except when very small areas (such as one city lot) are being considered. It is only at these very small scales that the mean surface slope and actual area and length come close to fitting the basic theoretical concept.

The following data are needed as input to HEC-1 to describe each overland flow strip:

a. L_o - typical overland flow length

b. S_o - representative slope

c. N - roughness coefficient (see Table 2.1)

d. A_{o_1}, A_{o_2} - the percentages of the subbasin area which the overland flow surface represents (two possible types for each subbasin)

e. Infiltration and loss rate parameters

Ways to determine these parameters are discussed in the following section. It is suggested that the data first be tabulated on a form for the entire basin and then entered on the coding sheet used for computer program data preparation. A typical data tabulation sheet is given in Appendix B.

Selection of Overland Flow Parameters

The area is the simplest quantity to specify; the area of each element is given as a percentage of the total area of the subbasin. If a single element is used, one hundred percent is specified. If two elemerts are used, the sum of the percentages should be one hundred percent.

The slope is a value which is representative of the slope of the path that the water takes on its way to the collector channel. It may differ from the mean topographic slope for the catchment, and it is usually strongly related to the type of land use or development. For an urban setting, a single slope-value could be used for all areas of

similar building practice, even though the mean ground slopes vary significantly.

As discussed in Chapter 1, the kinematic wave equations for the wide flow planes of the overland flow elements are based on Manning's equation for flow in a wide channel:

$$q_o = \alpha_o y_o^{m_o} \qquad 2.1 \qquad \text{(See Eq 1.11)}$$

where:

q_o = flow per unit width

y = flow depth

α_o = kinematic wave parameter

m_o = kinematic wave parameter

For this situation:

$$m_o = 5/3$$

$$\alpha_o = \frac{1.49}{N} S_o^{1/2}$$

where:

S_o = slope

N = surface roughness coefficient similar to Manning's 'n' for channel flow.

Because the nature of sheet flow with very small depths over rough surfaces differs markedly from streamflow, these roughness coefficients have much different values than the Manning's 'n' values used in streamflow computations. Values of N found to be appropriate are given in Table 2.1.

TABLE 2.1

Catchment Roughness Parameters for Overland Flow

Surface	N
Dense Growth*	0.4 - 0.5
Pasture*	0.3 - 0.4
Lawns*	0.2 - 0.3
Bluegrass Sod**	0.2 - 0.5
Short Grass Prairie**	0.1 - 0.2
Sparse Vegetation**	0.05 - 0.13
Bare Clay-Loam Soil (Eroded)**	0.01 - 0.03
Concrete/Asphalt - Very Shallow Depths* (depths less than 1/4 in)	0.10 - 0.15
- Small Depths* (depths on the order of 1/4 in to several inches)	0.05 - 0.10

* from Crawford and Linsley (1966)
**from Woolhiser (1975)

A critical parameter in the overland flow element description is the flow length L_o. A proper specification of L_o is vital, since it is the most important parameter in determining the response characteristics of the overland flow elements. The overland flow length can be looked on as the maximum length of the path taken by a representative water drop to reach a channel where it first moves as streamflow. It is thus the distance for overland flow to reach a tributary or local channel, such as a street gutter.

Fortunately, in many natural basins and urban catchments, close examination of the full drainage system reveals that the small-scale drainage patterns are quite similar throughout the entire basin. The value of L_o appropriate for such a situation will not vary greatly over the basin. The actual values of L_o which give the correct run-off response for the basin must be verified through comparison of model output with measured data, however.

Collector Channel

The collector channel element is used to model the flow in its path from the point where it first becomes channel flow to the point where it enters the main channel. The inflow to the collector channel is taken as a uniformly distributed flow along the entire length of the channel. This correctly represents the situation where overland flow runs directly into the gutter, and also provides a reasonable approximation of the flow inputs into the storm drain system from individual catch basins and tributary collector pipes which are distributed along the collector channel.

The kinematic wave equations (developed in Chapter 1) are the continuity equation for unsteady channel flow with lateral inflow, and Manning's equation.

The continuity equation is:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x} = q_o \qquad (2.2) \quad \text{(See Eq 1.14)}$$

where, Q_c is the channel flow in cfs, A_c is the flow cross sectional area in ft^2, q_o is lateral inflow in cfs/ft to the channel, x is distance

along the channel in ft, and t is time in seconds. Manning's equation is written in the form:

$$Q_c = \alpha_c A_c^{m_c} \tag{2.3}$$

where the kinematic wave routing coefficients α and m are a function of the channel geometry. The general expression for α is:

$$\alpha_c = \frac{KS_c^{1/2}}{n} \tag{2.4}$$

where K is a constant that depends on the channel geometry, S_c is the channel slope, and n is Manning's roughness coefficient. A change in either n or S_c will change the value of α_c used in the calculations. It should be noted that the model is more sensitive to changes in n than to S_c because α_c depends on the first power of n while it is proportional to the square root of S_c.

The value of the exponent m_c for trapezoidal channels ranges from 4/3 when the trapezoid has a base width of zero (triangular shape) to 5/3 for a very wide rectangular shape. For a channel with a circular cross section, m_c is taken as 1.25. (See Chapter 1 for the derivation of these parameters.)

The following data are needed as input to describe the collector channel system (refer to Figure 1.7):

1. The surface area drained by a single representative collector channel (e.g., gutter plus storm drain), A_c.

2. The collector channel length (total length of gutter plus length of storm drain), L_c.

3. The channel shape (either a circular section or some variant of a trapezoid).

4. The pipe diameter or the trapezoid bottom width and side slope, if appropriate.

5. The channel slope, S_c and

6. Manning's 'n'.

Selection of Collector Channel Parameters

The characteristics of the collector channel components can be determined by looking at a drainage map for the basin and selecting a typical collector system for each subbasin. This single typical collector channel is used to represent all the collector channels in the subbasin.

1. The area associated with the collector system can be determined from the map. This is an area in square miles (sq. km.) rather than the percentage of the subbasin area. It does not have to be an integer multiple of the subbasin area.

2. The collector channel length is taken as the longest flow path from the upstream end of the collector system to its outlet at the main channel. This length should include the distance the water will travel as gutter flow.

3. The channel shape and size will usually change along the length of the channel; however, a single shape must be chosen to represent the channel along its entire length. This is not as great a problem as might appear at first. As shown in Figure 2.4, a triangle with side slope of one to one matches reasonably well the area-discharge relationships for circular conduits for a given slope and roughness. The triangular shape is the one used by the computer if the shape is not specified in the input data. The selection of channel shape is discussed in more detail in the section on the main channel, below.

4. If the representation is by a circular or trapezoidal shape, the channel dimensions chosen should represent the most commonly used size of channel in the system.

5. The channel slope can be estimated from a topographic map by taking the difference in elevation between the upstream and downstream ends and dividing by the length. If drop structures are used in the storm drains, the slopes should be adjusted accordingly.

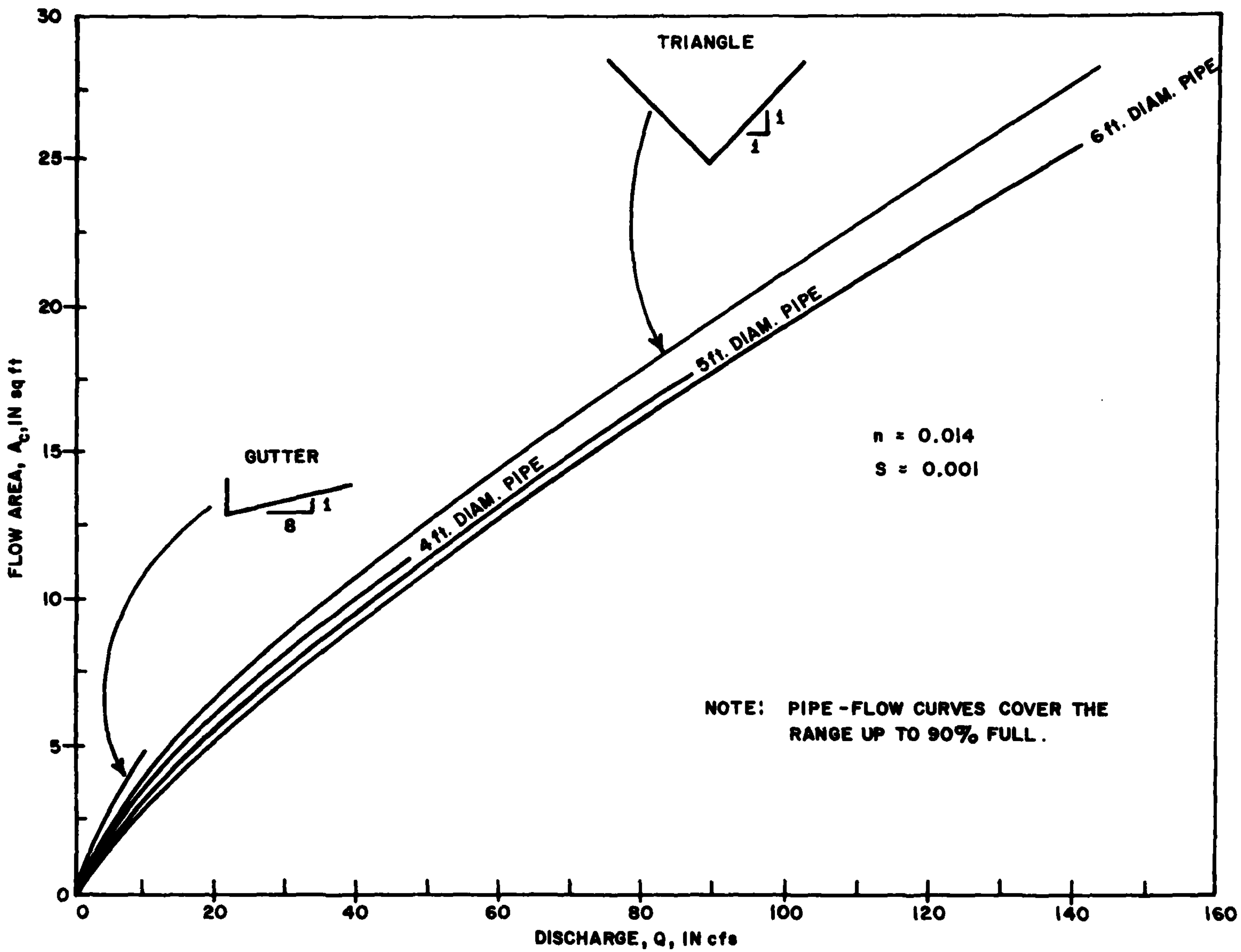


Figure 2.4 Discharge Versus Flow Area for Various Cross Sections

6. A Manning's 'n' which best represents the roughness of the major portion of the channel should be used. Tables of 'n' for various types of channels, such as concrete pipe, lined or unlined open channels, etc., are available in hydraulic handbooks and other sources.

Main Channel

The main channel can carry both inflows from upstream subbasins as well as flows supplied by the collector channels within the subbasin. The inflow from the collector channel is taken to be uniformly distributed along the length of the main channel. This is assumed to reasonably approximate the actual situation where the flow enters the channel from the various collectors at a number of discrete points at various spacings. In Equation 2.2, the lateral inflow, q_c, is determined by scaling up the collector channel flow to match the total subbasin areas and then dividing the flow by the total main channel length. For example, suppose that the subbasin area is 1.0 square mile, while the collector channel area is 0.30 square mile and the length of the main channel is 2,000 feet. If the collector channel flow is designated as Q_c, the inflow per foot of main channel is

$$q_c = (1.0/0.30)(Q_c/2000) \text{ cfs/ft}$$

The channel routing element can also be used independently for routing a hydrograph through a channel reach. If desired the subbasin flow can be computed separately and combined with routed flow at the subbasin outlet. Any of the routing methods available in HEC-1 can be used for channel routing (Muskingum, modified Puls, Tatum, etc.) if desired.

The channel routing procedure requires the following data:

1. Channel or stream length, L_m
2. Slope, S_m
3. Manning's 'n'
4. Area of subbasin, $A_{subbasin}$
5. Channel shape (trapezoidal or circular)
6. Channel dimensions (e.g., width, w, or diameter, D_c, if required, and side slopes, z)
7. The upstream hydrograph to be routed through the reach if desired.

Selection of Main Channel Parameters

Most of the channel data can be obtained from physically measurable parameters for the channel and subbasin, with exception of Manning's 'n'. The following procedure can be used to determine these data:

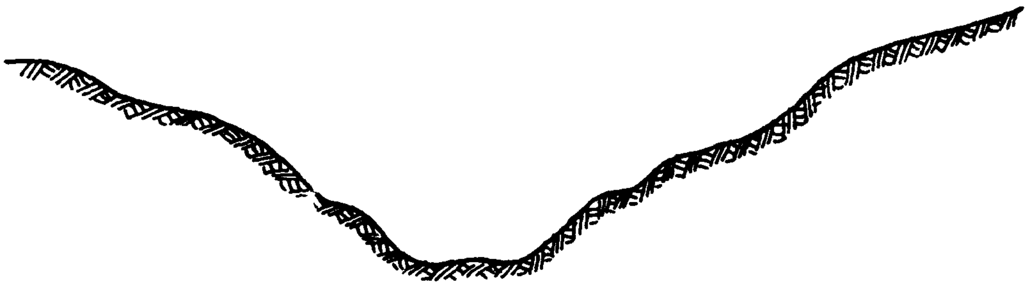
1. The channel length can be scaled from a drainage map of the basin.
2. The mean channel slope can be obtained from field measurements or estimated using topographic maps.
3. Selection of Manning's 'n' should be based on the average channel conditions.
4. The subbasin area can be measured from topographic maps.
5. The selecting of channel cross section is discussed in the following section.
6. The channel dimensions follow from the preceding item.
7. An upstream hydrograph will not be routed through the channel reach unless the user specifically requests the program to do so.

Selection of Channel Cross Section

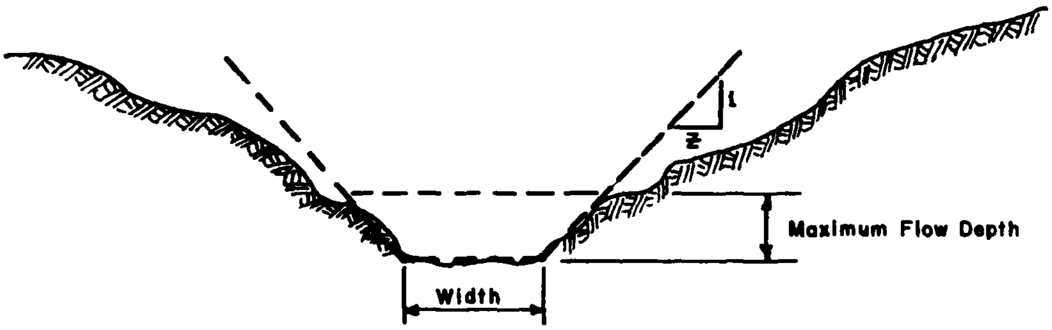
The channel cross section for either the main channel or the collector channel can be defined as one of two simple shapes to permit modeling a variety of natural channels. The kinematic wave model is not especially sensitive to channel cross-sectional shape in the simulation of discharge, and therefore, it is not necessary to use complex channel shapes. The shapes which can be used in HEC-1 are trapezoidal and circular. Most main channels can be best simulated by a triangular shape. This can be done by specifying the base width for the trapezoid as zero, or for a collector channel, by using the default values. When the channel is small, flood flows generally require overbank areas to carry the flows, and a triangular shape usually represents this situation quite well. As seen by a plot of area versus flow, Figure 2.4, a triangle can be used to approximate circular sections as well as street gutters.

In the downstream reaches of the basin, the trapezoidal section is usually the most appropriate for open channels. The shape should provide the best fit to the channel shape for the given flood flow, however. For example, in some situations a trapezoid might be the best fit for low flows, while a triangle might be more appropriate for high flows. An illustration of this is shown in Figure 2.5.

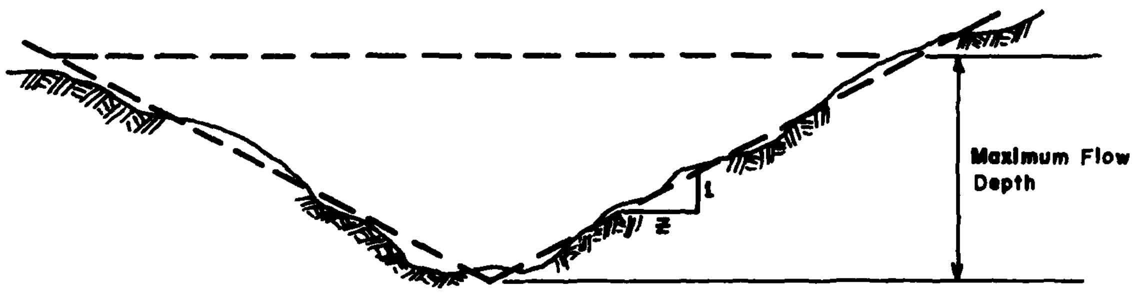
The circular section allows modeling of storm sewers. The flow behavior of the conduit is simulated properly up to the point where the conduit is approximately ninety percent full. The program does not



a) Channel Cross Section



b) Representation by a Trapezoid



c) Representation by a Triangle

Figure 2.5 Fitting Channel Cross Sectional Shape

handle the effects of pressure flow, and for flows greater than about ninety percent of pipe capacity, the program assumes that the capacity of the element increases as required and has no upper limit. In many cases this approach is an adequate representation of what is happening in nature, because water that does not enter the storm drains flows over the surface or along a roadway until it finds another location to enter the drainage system.

AN EXAMPLE APPLICATION OF KINEMATIC WAVE METHODS

The small partially-urban basin shown in Figure 2.6 is to be modeled using kinematic wave runoff and routing options of HEC-1. The hydrologic characteristics of this basin (obtained by previous calibration) are as follows:

Subbasin 1. The upper of the two subareas making up the basin is presently not urbanized and is primarily rolling pasture land with few trees. The typical distance L_o for flow to travel to tributary stream channels is 500 feet. The overland flow roughness coefficient N is 0.4. The representative ground slope S_o is 0.04. The amount of impervious area is assumed to be negligible. The subbasin area A_{o_1} is 1.5 square miles.

The collector or tributary channels have a typical slope S_c of 0.025, and an 'n' value of 0.10, with a typical channel length, L_c, of 1,500 feet. The most representative section is a triangle. The area, A_c, contributing to a typical collector stream is 0.4 square mile.

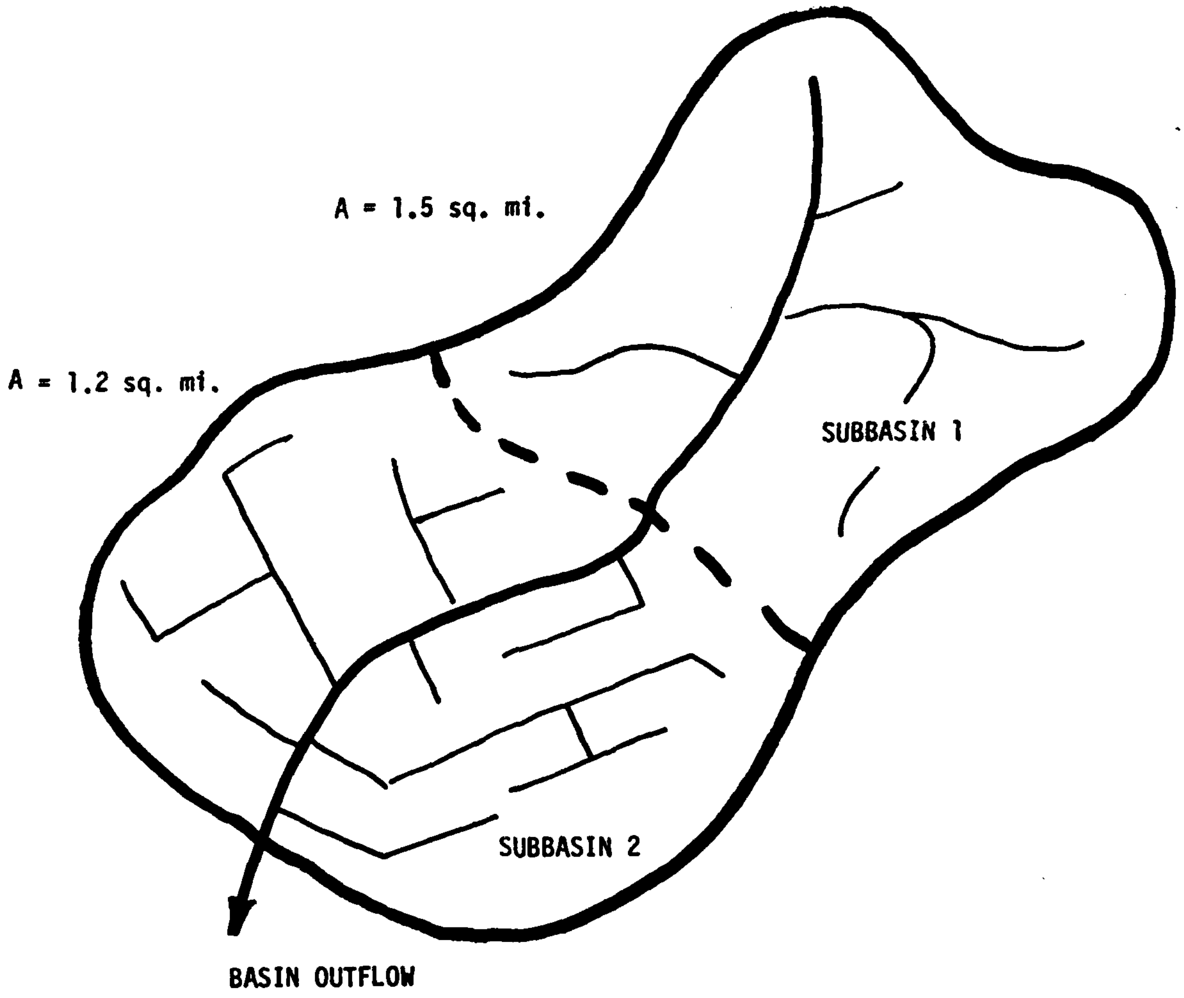


Figure 2.6 Basin for Example Problem

The main channel is approximately triangular in cross section with side slopes z of 1 in 4. The mean channel slope S_m is 0.01 and Manning's 'n' is 0.05. Its length, L_m, is 3,500 feet.

Subbasin 2. The lower subbasin is completely urbanized, and twenty percent of the subbasin surfaces are impervious. In this subbasin the impervious runoff areas have the following characteristics: L_{o_1} = 50 feet, S_{o_1} = 0.06, N = 0.15. The pervious areas can be represented by the following parameters: L_{o_2} = 130 feet, S_{o_2} = 0.01, N = 0.3. The subbasin area, A_{o_2}, is 1.2 square miles. The total basin area is 2.7 square miles.

The collector channel system involves three hundred feet of gutter plus an additional 1,800 feet of pipe storm drain ranging up to four feet in diameter. A triangular section is used to represent the various channel components (the program default value with one to one side slopes). The average slope, S_c, is 0.008, and the Manning's 'n' which accounts for friction and other channel head losses is 0.020. The area, A, contributing to the collector channel system is 0.35 square mile.

The parameters describing this basin are given in Table 2.2. Also, a listing of the program input for modeling this basin is provided in Table 2.3, and hydrographs from the run output are shown in Figure 2.7.

As an example of the way the program can be used to evaluate the effects of future urbanization, the following changes were made to the parameters describing Subbasin 1. Two overland flow strips were used instead of one; an impervious overland flow element with L_{o_1} = 50

TABLE 2.2

Parameters for Kinematic Wave Example Problem

	% Total Subbasin Area	L ft	S	Roughness	Channel Shape	Channel Size ft	Z	Collector System Area mi^2	Loss Rate in/hr
Subbasin 1 (1.5 mi^2)									
1. Overland Flow Strip (101)	100	500	0.04	0.40	--	--	-	--	0.50
2. Collector Channel (10)	--	1,500	0.025	0.10	Triangle	--	1	0.40	--
3. Main Channel (1)	--	3,500	0.010	0.05	Triangle	--	4	--	--
Subbasin 2 (1.2 mi^2)									
1. Overland Flow Strip 1 (201)	20	50	0.06	0.30	--	--	-	--	0.02
2. Overland Flow Strip 2 (202)	80	180	0.01	0.40	--	--	-	--	0.20
3. Collector Channel (20)	--	2,100	0.008	0.020	Triangle	--	1	0.35	--
4. Main Channel (2)	--	4,000	0.003	0.025	Trapezoid	2	2	--	--

TABLE 2.3

LISTING OF INPUT DATA FOR EXAMPLE PROBLEM

```
LINE        ID.......1.......2.......3.......4.......5.......6.......7.......8.......9......10

   1        ID              KINEMATIC WAVE ROUTING IN HEC-1
   2        ID        EXAMPLE BASIN MODEL USED IN TRAINING DOCUMENT 10
   3        ID      INPUT DATA FORMATS REVISED FOR HEC-1N  ---- AUG 1981
   4        ID                      ++++++++++
   5        ID                ORIGINAL BASIN CONDITIONS

   6        IT       5  1MAY79    1200     100
   7        IO       4

   8        KK    SUB1
   9        KM       SUBAREA 1   1.5 SQ MI  L = 3500 FT
  10        KO       3       2
  11        PB     2.0
  12        PI       6      12      15      25      20      10       8       4
  13        LU     0.0    0.50     0.0
  14        BA     1.5
  15        UK     500     .04      .4     100
  16        RK    1500    .025     .10      .4
  17        RK    3500     .01     .05    2.17    TRAP       0       4

  18        KK    SUB2
  19        KM       SUBAREA 2  1.2 SQ MI  L = 4000 FT
  20        KO       1       2
  21        LU     0.0    0.02     0.0     0.0    0.20     0.0
  22        BA     1.2
  23        UK      50     .06      .3      20
  24        UK     180     .01      .4      80
  25        RK    2100    .008     .02     .35
  26        RK    4000    .003    .025       0    TRAP       2       2     YES
  27        ZZ
```

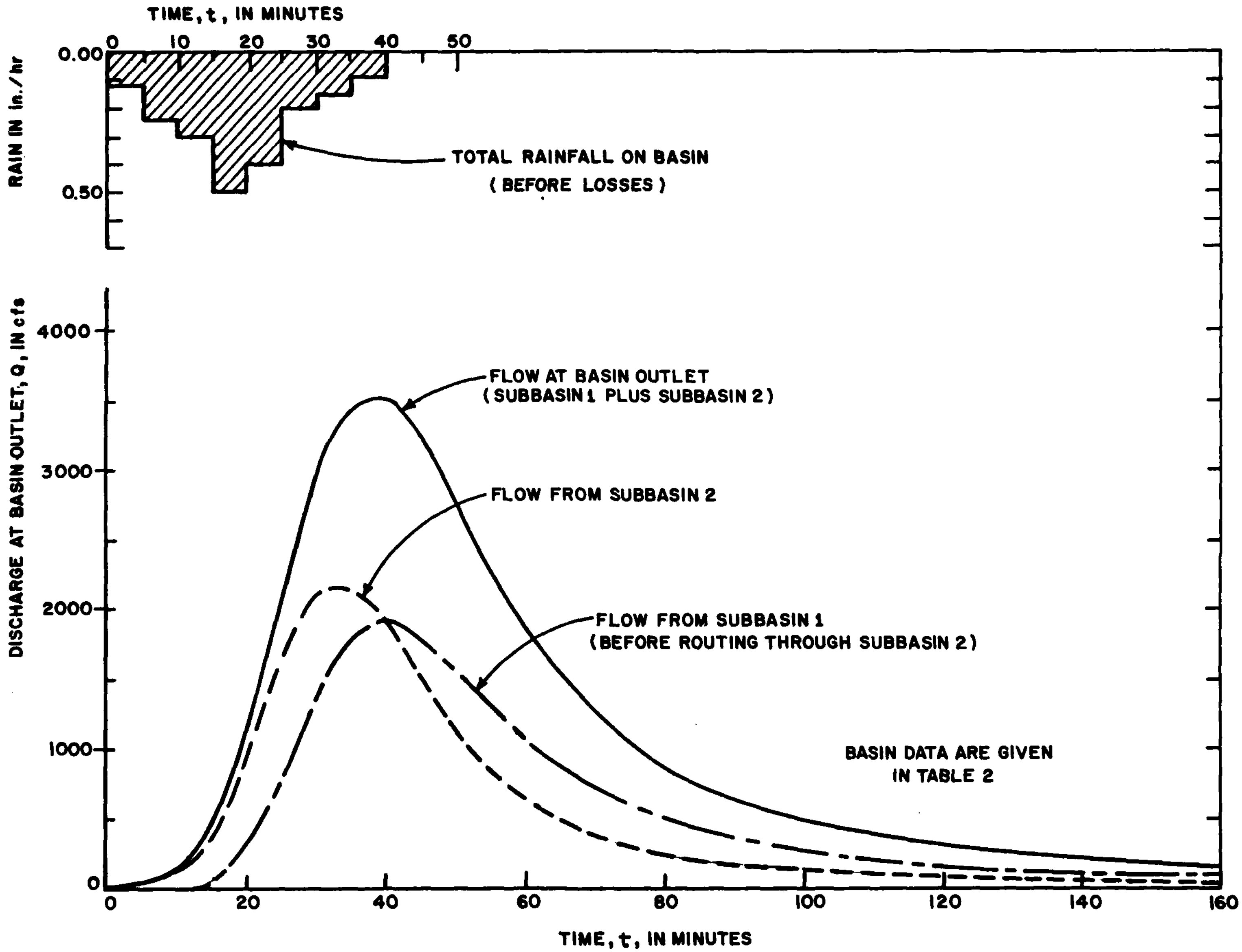


Figure 2.7 Computed Hydrographs for Example Problem

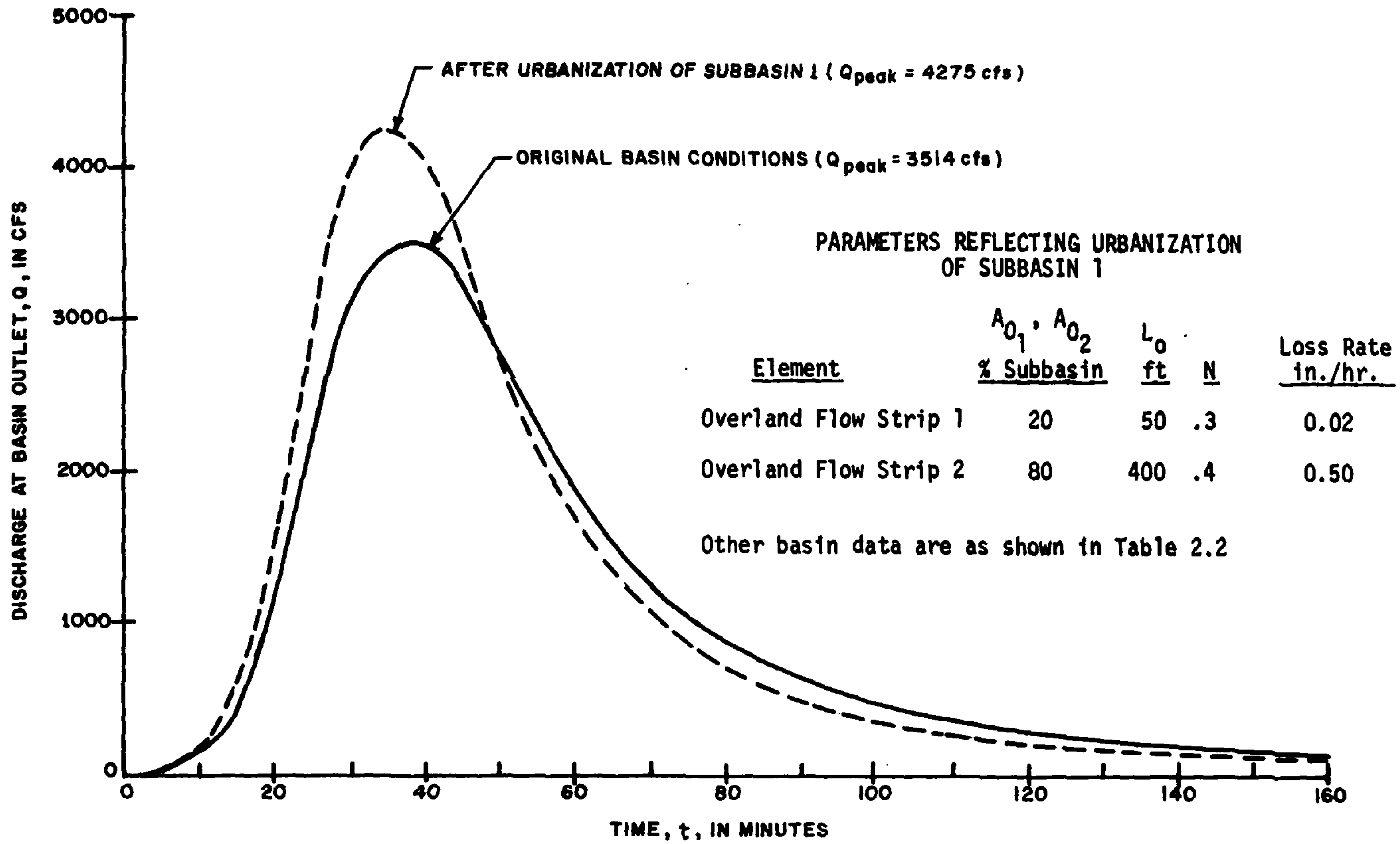
feet, N = 0.3, and representing twenty percent of the subbasin area; and a pervious element with the representative flow lengths reduced to four hundred feet and with the other parameters as before. The results from this run are plotted in Figure 2.8. The peak flow at the basin outlet increased from 3,514 cfs for the initial basin condition to 4,275 cfs after the urbanization changes. The peak flow occurred five minutes earlier in the fully urbanized basin.

PARAMETER SENSITIVITY

Additional runs were made with changes to some of the parameters describing the above basin to give an indication of the sensitivity of the modeling process to changes of various magnitudes. The pertinent features of the runs (in which the overland flow length and overland flow roughness parameters were adjusted) are given in Table 2.4. Only Subbasin 2 was modeled in these runs, and therefore the tabulated peak flows are lower than those given above for the full basin.

As shown in Table 2.4, increasing the overland flow length reduces the computed peak flow, and the peak occurs later. Using short lengths has the opposite effect. These results are shown in graphical form in Figure 2.9.

The effect of changes to the surface roughness parameter is illustrated in the second part of Table 2.4 and in Figure 2.9. Low values of N produce high computed flows with short times to the flow peak, while high N values retard the flow and give lower peak values.



Element	A_{0_1}, A_{0_2} % Subbasin	L_o ft	N	Loss Rate in./hr.
Overland Flow Strip 1	20	50	.3	0.02
Overland Flow Strip 2	80	400	.4	0.50

Figure 2.8 Effect of Urbanization on Basin Outflow

TABLE 2.4

Sensitivity to Changes in Parameters[1]

a. Effect of Overland Flow Length

Case	L_{202}[2] ft.	Q_{peak}[3] cfs	Time of Peak min
A1	90	2,749	30
A2	135	2,558	30
A3	150	2,413	30
A4	180	2,163	30
A5	225	2,007	35
A6	270	1,782	35

b. Effect of Surface Roughness

Case	N_{201}[4]	N_{202}[5]	Percent of Original N Values	Q_{peak}[2] cfs	Time of Peak min
B1	0.15	0.20	50	2,734	30
B2	0.22	0.30	75	2,551	30
B3	0.30	0.40	100	2,163	30
B4	0.45	0.60	150	1,793	35
B5	0.60	0.80	200	1,499	35

[1] With the exception of the changes listed here, all parameters describing Subbasin 2 are as given in Table 2.2.

[2] L_{202} is overland flow strip number 2 for subbasin 2.

[3] This is the computed peak flow from Subbasin 2 only.

[4] N_{201} is roughness coefficient for overland flow strip number 1 for subbasin 2.

[5] N_{202} is roughness coefficient for overland flow strip number 2 for subbasin 2.

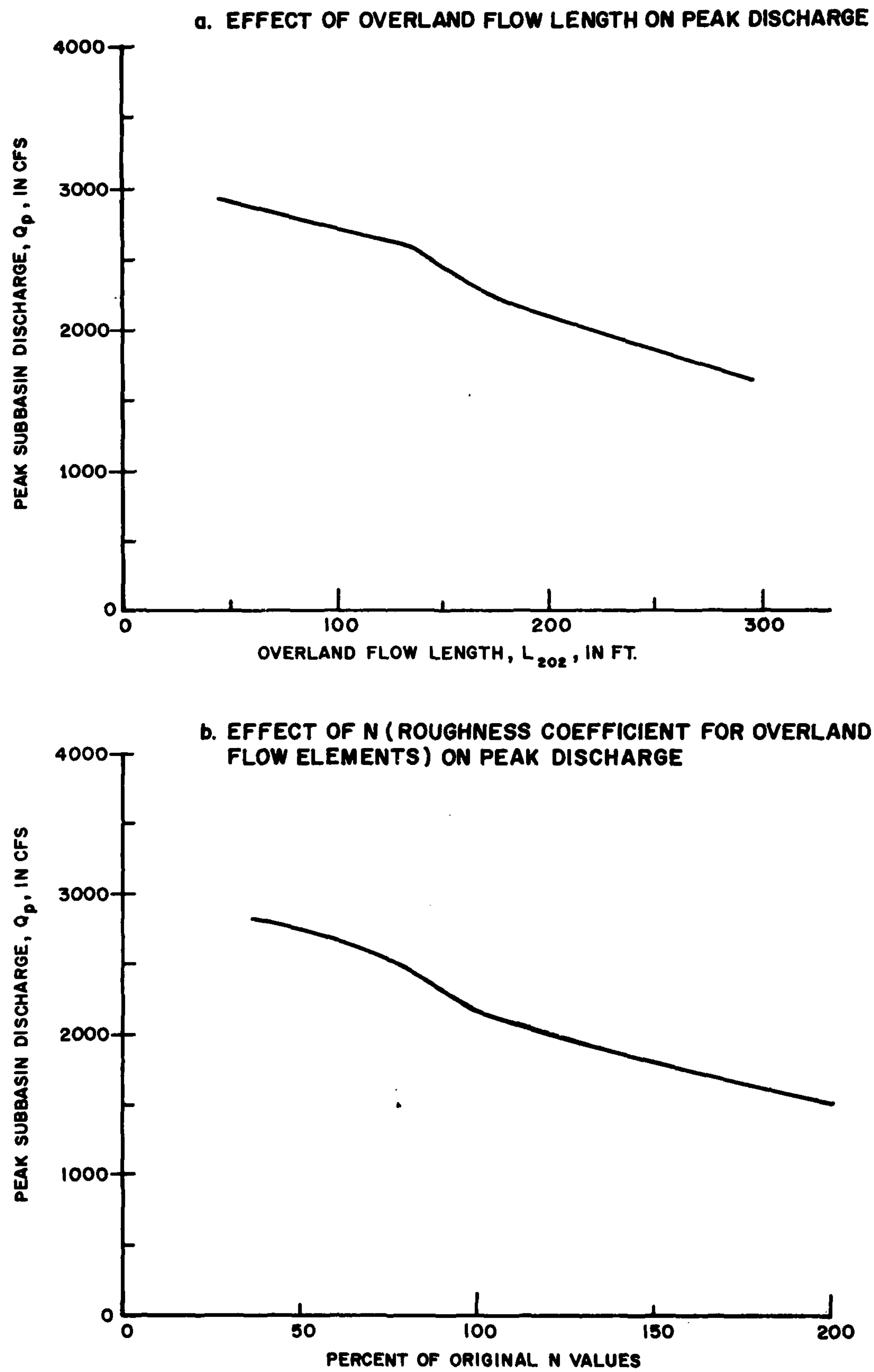


Figure 2.9 EFFECTS OF CHANGES TO KINEMATIC ROUTING PARAMETERS

Another kinematic wave routing parameter which can be adjusted is the slope. However, both the slope, S_0, and the roughness, N, are combined in the parameter α_0, and either may be adjusted to produce the desired effect. However, as indicated by Equation 2.4, α_0 is a function of $S_0^{1/2}$; therefore, changing S_0 has less relative effect than changing the roughness. For example, to produce the same change in α_0 as caused by reducing N by a factor of two, the slope, S_0, would have to be four times as large.

The effect of changes to the channel routing parameters is illustrated here by showing the influence of the channel roughness on the hydrograph from Subbasin 2 alone (Subbasin 1 flow is not considered). The outflow hydrographs for two different 'n' values are shown in Figure 2.10, and data for other 'n' values are given in a table in Figure 2.10. In this case, changes to the channel roughness have a smaller effect on the hydrograph than changes to the overland flow parameters.

SUMMARY AND CONCLUSIONS

The material presented in this paper provides the user of computer program HEC-1 with background information for modeling hydrologic basins using kinematic wave routing. The example problem gives an illustration of the use of the method and shows how it can be applied to studies of the effects of urbanization on hydrologic basins. Some guidance on selection of parameters is provided, along with a brief sensitivity analysis to show effects of varying the parameters.

Effect of "n' on Peak Flow

Manning's 'n'	Q_{peak} cfs	t peak min
0.020	2202	30
0.025	2165	30
0.030	2140	35
0.040	2115	35
0.050	2079	35

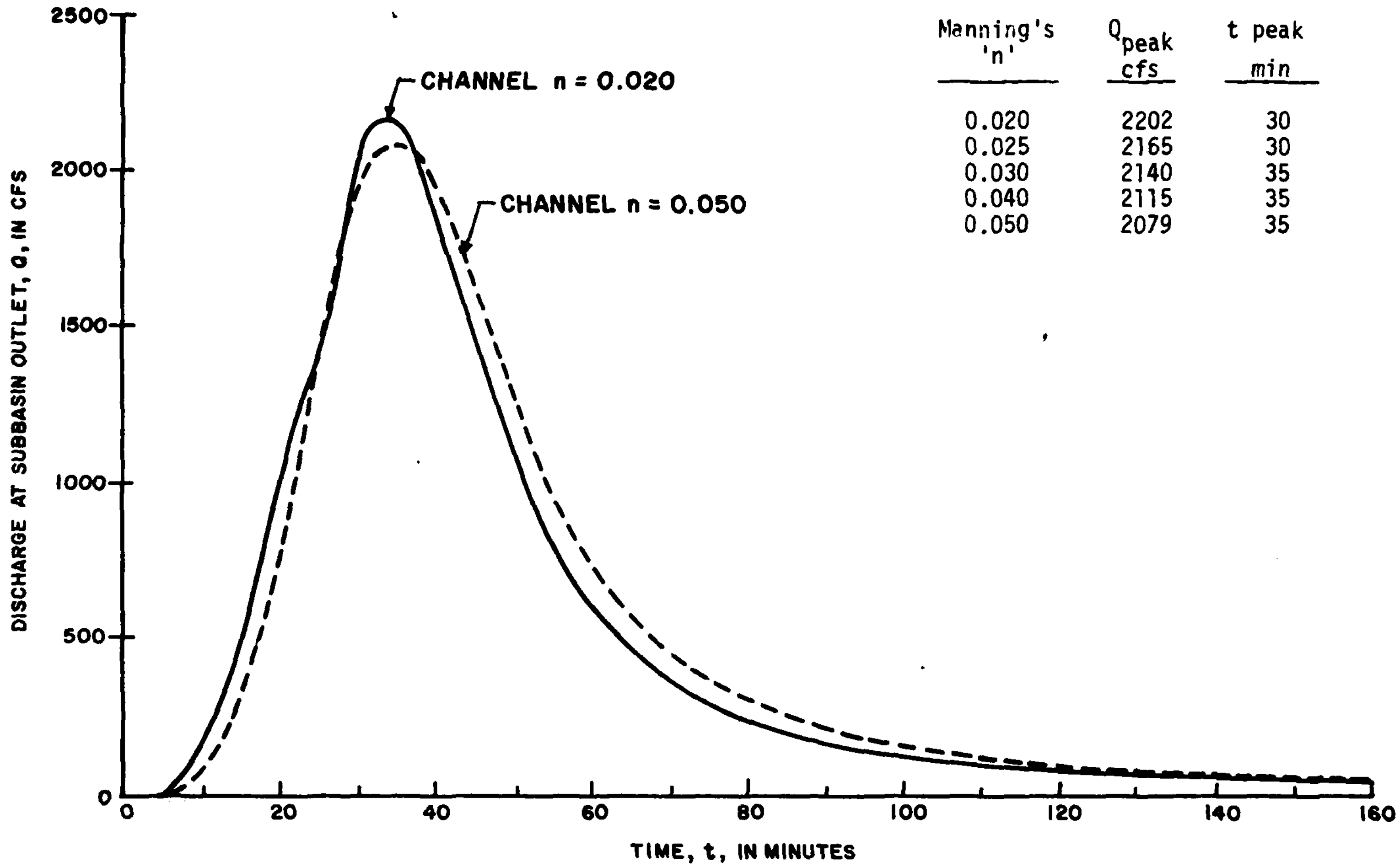


Figure 2.10 Effects of Channel Roughness on Basin Response

It is important for the user of this method to verify his model using measured rainfall-runoff events to allow the assessment of the perfomance of the model with the selected parameters. Without such a check the inexperienced modeler should interpret his results with a great deal of caution. However, kinematic wave models have been used successfully in urban hydrology in a large number of applications, and when the models are properly formulated, good simulations result.

ACKNOWLEDGMENTS

The basic approach for the subdivision of the basin into the various elements listed here was developed and coded through discussions with John Peters, Art Pabst and Paul Ely of the Hydrologic Engineering Center. Wayne Pearson assisted in the calculations used in the examples found in Chapter 2.

The development and programming of the kinematic wave routing routines in computer program HEC-1 was done under contract to the Hydrologic Engineering Center by Resource Analysis, Incorporated, under the direction of Dr. Brendan Harley.

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APPENDIX A

EXAMPLE OF KINEMATIC WAVE ROUTING BY HAND CALCULATIONS

To illustrate how the kinematic wave calculations are carried out in HEC-1, the following simple example is presented. In it, the results of hand calculations are given, along with the equations used. A computer run was also made using the same data and the computer results were compared with the hand calculations.

The data for the problem are given in Table A.1. The overland flow element is taken as a single rectangular plane with a trapezoidal channel running along its lower edge. The time increment used in this simulation is five minutes. Rainfall is assumed to occur at the rate of one inch per hour in the first five minutes, at two inches per hour in the next five minutes, and at one inch per hour the next five minute period. No rain occurs after fifteen minutes.

The equations used for kinematic wave routing are presented in Chapter 1. Two different forms are used: (1) the "Standard Form" for situations when the average wave celerity $\bar{c}$ is less than the ratio of the distance increment to the time increment $\Delta x/\Delta t$; and (2) the "Conservation Form" for cases where $\bar{c}$ is greater than $\Delta x/\Delta t$. The value of $\bar{c}$ is based on an average representative flow area for the entire channel reach, $\bar{A}$, and is computed from the equation

$$\bar{c} = \alpha m (\bar{A})^{m-1} \tag{A.1}$$

where $\bar{c}$ is the representative wave speed for the reach.

TABLE A.1

DATA FOR EXAMPLE

Overland Flow Element

Dimensions: 50 ft x 4,000 ft

Overland Flow Length = 50 ft

Slope = 0.06 ft/ft

Roughness Coefficient = 0.3

Loss Rate = 0 (Entirely impervious)

Channel Element

Length = 4,000 ft

Slope = 0.003 ft/ft

Roughness ('n') = 0.025

Shape: Trapezoidal

Bottom Width = 2 ft

Side Slope: 2 to 1

Time Increment = 5 min

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Standard Form of Kinematic Wave Equations ($\bar{c}$ less than $\Delta x/\Delta t$)

The subscripts i and j associated with a given variable designate distance and time respectively. For example, if $A_{i,j}$ is the numerical value of A at time t and location i, then at time $t + \Delta t$ and the same location, the variable is designated as $A_{i,\, j+1}$. The location Δx downstream is designated by the subscript i+1. The reader should refer to Figures 1.8 and 1.9 in Chapter 1, where details of finite difference approximations were developed. (The equation below, was developed in Chapter 1.)

The area at succeeding points along the channel in the downstream direction is computed from the equation

$$A_{i,j} = \bar{q}\Delta t + A_{(i,j-1)} - \alpha m\frac{\Delta t}{\Delta x}\left[\frac{A_{(i,j-1)} + A_{(i-1,\, j-1)}}{2}\right]^{m-1}\left[A_{(i,\, j-1)} - A_{(i-1,\, j-1)}\right] \quad (1.32)$$

and the flow is computed from

$$Q_{i,j} = \alpha(A_{i,j})^m \quad (1.14)$$

where the symbols are as defined above. For overland flow computations, q is the effective rainfall (rainfall minus infiltration losses) per square foot of surface. For channel flow calcuations q is the lateral inflow per foot of channel.

Conservation Form of the Equations ($\bar{c}$ greater than $\Delta x/\Delta t$)

The equations used in this case are

$$Q_{i,j} = Q_{(i-1,\, j)} + \bar{q}\Delta x + \frac{\Delta x}{\Delta t}\left[A_{(i-1,\, j)} - A_{(i-1,\, j-1)}\right] \quad (1.36)$$

and

$$A_{i,j} = (Q_{i,j}/\alpha)^{1/m} \tag{1.37}$$

<u>Results</u>

The results of the computations are given in Table A.2 for the overland flow process and Table A.3 for the channel flow. The results of the hand computations agreed with the computer results to within the limits of precision of the hand calculations.

TABLE A.2

OVERLAND FLOW CALCULATIONS

j	t min	RAINFALL q cfs/ft^2	FORM OF EQUATIONS*	$A_{1,j}$ ft^2	$A_{2,j}$ ft^2	FLOW TO CHANNEL $Q_{2,j}$ cfs/ft
0	0			0	0	0
		0.000 0231	S			
1	5			0.00694	0.00694	0.000 308
		0.000 0463	C			
2	10			0.0154	0.0188	0.001 61
		0.000 0231	C			
3	15			0.0101	0.0186	0.001 59
		0.0	C			
4	20			0.0	0.0127	0.000 844
		0.0	S			
5	25			0.0	0.00207	0.000 041
		0.0	S			
6	30			0.0	0.00155	0.000 025
		0.0	S			
7	35			0.0	0.00123	0.000 017

$\Delta x = 25$ ft $L_o = 50$ ft

$\Delta t = 5$ min $= 300$ sec No. $\Delta x = 2$

$\alpha_o = 1.22$ Total Area $= 80{,}000$ ft^2

$m_o = 1.667$

*S designates Standard Form of Equations were used

C designates Conservation Form of Equations were used

TABLE A.3

CHANNEL FLOW COMPUTATIONS

j	t min	q* cfs/ft	FORM OF EQUATION**	$A_{1,j}$*** ft^2	$A_{20,j}$ ft^2	CHANNEL OUTFLOW $Q_{20,j}$ cfs
0	0	0		0	0	0
1	5	0.000 308	C	0.050	0.092	0.057
2	10	0.001 61	C	0.161	0.566	0.658
3	15	0.001 59	C	0.167	0.973	1.369
4	20	0.000 844	C	0.105	1.063	1.542
5	25	0.000 041	C	0.011	0.862	1.162
6	30	0.000 025	C	0.008	0.658	0.808
7	35	0.000 017	C	0.006	0.485	0.534

Δx = 80 ft L_m = 1,600 ft

Δt = 5 min = 300 sec   No. Δx = 20

α_m = 1.42

m_m = 1.35

* Corresponds to "Flow to Channel" in Table A.1

** S = Standard Form of Equation used

C = Conservation Form of Equation used

*** $A_{1,j}$ is 80 ft from upstream end of channel

APPENDIX B

SAMPLE DATA TABULATION FORM

Table B.1 can be reproduced and used for tabulating data describing the basin. The data can then be entered on a coding sheet for making an HEC-1 computer run.

TABLE B.1 KINEMATIC WAVE PARAMETERS - DATA TABULATION SHEET

Element	% of Total Subbasin Area	Loss Rate (in./hr.)	Length (feet)	Slope	Rough-ness	Channel Shape	Channel Size	Channel Side Slope	Area Served By Typ Coll (sq.mi.)
Subbasin Number ______									
(Area = ______ sq. mi.)									
Overland Flow Strip 1						—	—	—	—
Overland Flow Strip 2						—	—	—	—
Collector Channel	—	—							
Main Channel	—	—							—
Subbasin Number ______									
(Area = ______ sq. mi.)									
Overland Flow Strip 1						—	—	—	—
Overland Flow Strip 2						—	—	—	—
Collector Channel	—	—							
Main Channel	—	—							—
Subbasin Number ______									
(Area = ______ sq. mi.)									
Overland Flow Strip 1						—	—	—	—
Overland Flow Strip 2						—	—	—	—
Collector Channel	—	—							
Main Channel	—	—							—

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